Lipowsky *et al.* Reply: In our Letter[1] we introduced a multicomponent order parameter (OP) for surface melting and studied its behavior within phenomenological Landau models. The *minimal* set of OP components, $\mathbf{M}(\mathbf{Q}^{\parallel})$, as required by the translational invariance parallel to the surface, is parametrized by the reciprocal-lattice vectors $\mathbf{Q}^{\parallel}$ of the two-dimensional surface. Since the translational invariance perpendicular to the surface is broken, these OP's also depend on the distance z from the surface: $\tilde{M}=\tilde{M}(\mathbf{Q}^{\parallel},z)$. The OP components $\hat{M}(\mathbf{Q}^{\parallel},\mathbf{Q}^{\perp},z)$ considered by Löwen[2] satisfy

$$\tilde{M}(\mathbf{Q}^{\parallel},z)=\sum \hat{M}(\mathbf{Q}^{\parallel},\mathbf{Q}^{\perp},z)\exp(i\mathbf{Q}^{\parallel}\cdot\mathbf{e}_z z)\,, \tag{1}$$

where the summation runs over all reciprocal-lattice vectors $\mathbf{Q}^{\perp}$ such that $\mathbf{Q}=\mathbf{Q}^{\parallel}+\mathbf{Q}^{\perp}$ is a reciprocal-lattice vector of the three-dimensional bulk crystal.[3]

Quite generally, an interface or surface acts as a planar perturbation on any density (as long as the interfacial roughness can be ignored). For systems governed by short-range forces, such a perturbation decays exponentially $\sim\exp(-z/a)$ with the distance z from the interface but the decay length a depends on the density considered. Thus, within the liquid (or melt), the densities $\hat{M}(\mathbf{Q}^{\parallel},\mathbf{Q}^{\perp},z)$ and $\tilde{M}(\mathbf{Q}^{\parallel},z)$ are characterized by decay lengths $\hat{a}_L(\mathbf{Q}^{\parallel},\mathbf{Q}_{\perp})$ and $\tilde{a}_L(\mathbf{Q}^{\parallel})=\max_{\perp}[\hat{a}_L(\mathbf{Q}^{\parallel},\mathbf{Q}^{\perp})]$, respectively.

We found in Ref. 1 that the length scales $\tilde{a}_L(\mathbf{Q}^{\parallel})$ satisfy a simple algebraic relation within a square-gradient Landau model for the molten layer. It turns out, however, that such a relation holds for a *whole class* of Landau models which include gradients to arbitrary order as does the density-functional (DF) approach used by Löwen.[2] These Landau models are defined by

$$F_L\{M\}=\int d^2x\int_0^l dz\left\{\tfrac{1}{2}(\nabla M)^2+\sum_{m\geq 2}c_m M\nabla^{2m}M + \tfrac{1}{2}(M/a_{L0})^2-H_{\text{eff}}M\delta(z-l)\right\} \tag{2}$$

for the density $M(\mathbf{x},z)$ within the molten layer which is bounded by a vapor-liquid interface at $z=0$ and by a liquid-crystal interface at $z=l$. The effective field H_{eff} at the liquid-crystal interface represents the influence of the crystalline state on the molten layer and, thus, must reflect the translational symmetry parallel to the surface:

$$H_{\text{eff}}(\mathbf{x},z)=\sum \tilde{H}(\mathbf{Q}^{\parallel},z)\exp(i\mathbf{Q}^{\parallel}\cdot\mathbf{x})\,.$$

The model as given by (2) represents a generalization of Eq. (13) in Ref. 1.

Minimizing the above functional, one obtains

$$M(\mathbf{x},z)=\sum \tilde{M}(\mathbf{Q}^{\parallel})\exp[i\mathbf{Q}^{\parallel}\cdot\mathbf{x}-z/\tilde{a}_L(\mathbf{Q}^{\parallel})]\,, \tag{3}$$

with

$$\tilde{a}_L(\mathbf{Q}^{\parallel})=\tilde{a}_L(\mathbf{0})/\{1+[\tilde{a}_L(\mathbf{0})\mathbf{Q}^{\parallel}]^2\}^{1/2}\,, \tag{4}$$

where $\tilde{a}_L(\mathbf{0})$ depends on a_{L0} and on the coefficients c_m in (2).

On the other hand, the DF approach in Ref. 2 leads to length scales $\tilde{a}_L(\mathbf{Q}^{\parallel})$ which do not satisfy (4) for small values of $Q^{\parallel}$ even though they seem to approach this relation for large $Q^{\parallel}$. Furthermore, the scales for small $Q^{\parallel}$ are found to depend on the microscopic interparticle potential. In general, such a dependence, which cannot be obtained from phenomenological models as in (2), is indeed to be expected.

The length scales $\tilde{a}_L(\mathbf{Q}^{\parallel})$ determine the critical behavior of the surface OP's, $\tilde{M}(\mathbf{Q}^{\parallel},z=0)$. As pointed out in Ref. 1, all surface OP's *vanish simultaneously* at the melting temperature, $T=T_*$, *but with different rates.* For systems governed by short-range forces, the critical behavior is given by (Ref. 1)

$$\tilde{M}(\mathbf{Q}^{\parallel},z=0)\sim(T_*-T)^{\beta_1},\quad \beta_1=\frac{b\tilde{a}_L(\mathbf{0})}{\tilde{a}_L(\mathbf{Q}^{\parallel})}\,. \tag{5}$$

For the models considered in Ref. 1, the scales $\tilde{a}_L(\mathbf{Q}^{\parallel})$ satisfy the relation (4), $\tilde{a}_L(\mathbf{0})=a_{L0}$, and $b=1$ or $b=\frac{1}{2}$. These models have, however, one important limitation since the density $\tilde{M}(\mathbf{Q}^{\parallel}=\mathbf{0},z)$ which has the largest decay length is, in fact, *not* treated as an OP density (which would vanish at $T=T_*$) but rather as a *nonordering* density which stays finite at $T=T_*$.[4] This limitation is not present if one considers the densities $\hat{M}(\mathbf{Q}^{\parallel},\mathbf{Q}^{\perp},z)$ with decay lengths $\hat{a}_L(\mathbf{Q}^{\parallel},\mathbf{Q}^{\perp})$. Indeed, the largest decay length $\tilde{a}_L(\mathbf{0})$ can now belong to the nonordering density $\hat{M}(\mathbf{0},\mathbf{0},z)$ or to one of the OP densities $\hat{M}(\mathbf{0},\mathbf{Q}^{\perp},z)$ with $Q^{\perp}\neq 0$. The power-law behavior as in (5) still applies but three cases must be distinguished depending on the relative size of the two length scales $\kappa\equiv\hat{a}_L(\mathbf{Q}^{\parallel}=\mathbf{0},\mathbf{Q}^{\perp}=\mathbf{0})$ and $\xi_0\equiv\max_{\perp}[\hat{a}_L(\mathbf{Q}^{\parallel}=\mathbf{0},\mathbf{Q}^{\perp})]$, where the maximum is taken over all $Q^{\perp}\neq 0$. In terms of these scales, the coefficient b in (5) is given by $b=1$, $b=\kappa/\xi_0$, and $b=\frac{1}{2}$ for $\tilde{a}_L(\mathbf{0})=\kappa>\xi_0$, $\tilde{a}_L(\mathbf{0})=\xi_0>\kappa>\xi_0/2$, and $\tilde{a}_L(\mathbf{0})=\xi_0/2>\kappa$, respectively.

Reinhard Lipowsky
Sektion Physik der Universität München
Theresienstrasse 37
8000 München 2, West Germany

U. Breuer, K. C. Prince, and H. P. Bonzel
Institut für Grenzflächenforschung and Vakuumphysik
Kernforschungsanlage Jülich
5170 Jülich, West Germany

Received 21 August 1989
PACS numbers: 68.35.Rh, 64.70.Dv

[1] R. Lipowsky, U. Breuer, K. C. Prince, and M. P. Bonzel, Phys. Rev. Lett. **62**, 913 (1989).

[2] H. Löwen, preceding Comment, Phys. Rev. Lett. **64**, 2104 (1990).

[3] One should note, however, that the densities $\hat{M}(\mathbf{Q}^{\parallel},\mathbf{Q}^{\perp},z)$ are not uniquely defined by (1).

[4] R. Lipowsky, in Proceedings of the International Workshop on Magnetic Properties of Low-Dimensional Systems, San Luis Potosi, Mexico, Springer Proceedings in Physics (Springer-Verlag, New York, to be published).