

Morphological Complexity of Biomembranes and Vesicles

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- Membranes and vesicles
- Membrane tension and curvature elasticity
- Spontaneous curvature and membrane necks
- Morphological complexity

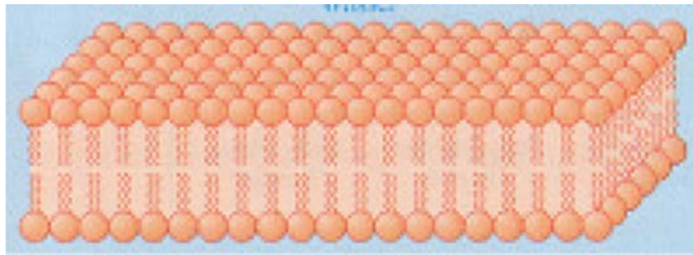
„Physical understanding from fruitful interplay between theory, simulation, and experiment“



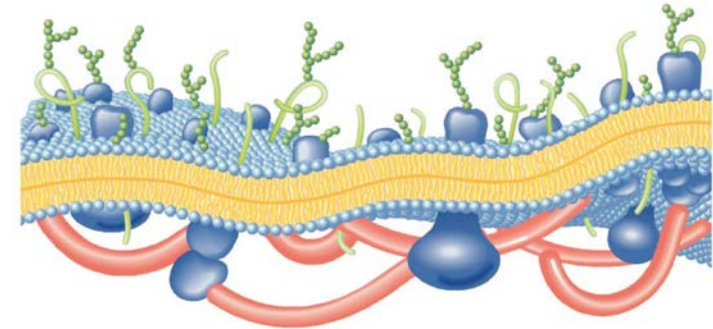
- Vesicles and Membranes
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- Spontaneous curvature and membrane necks
- Morphological complexity of vesicles

Multiscale Membranes

- Lipid bilayer

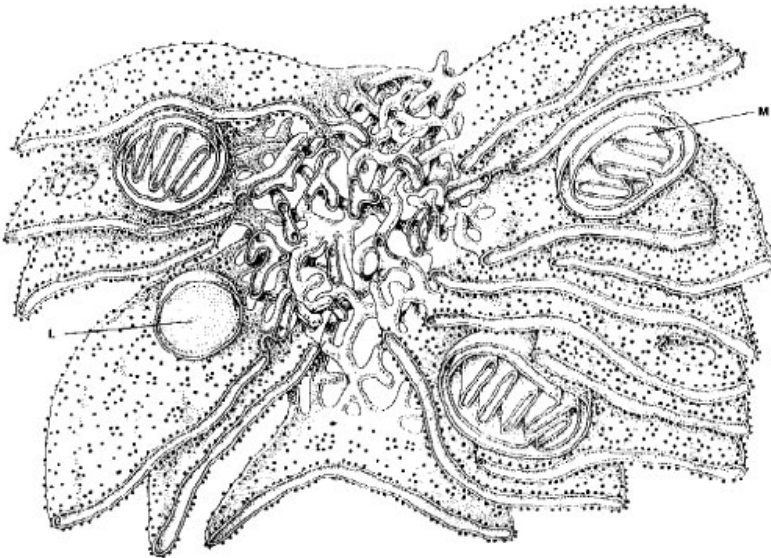


4 nm

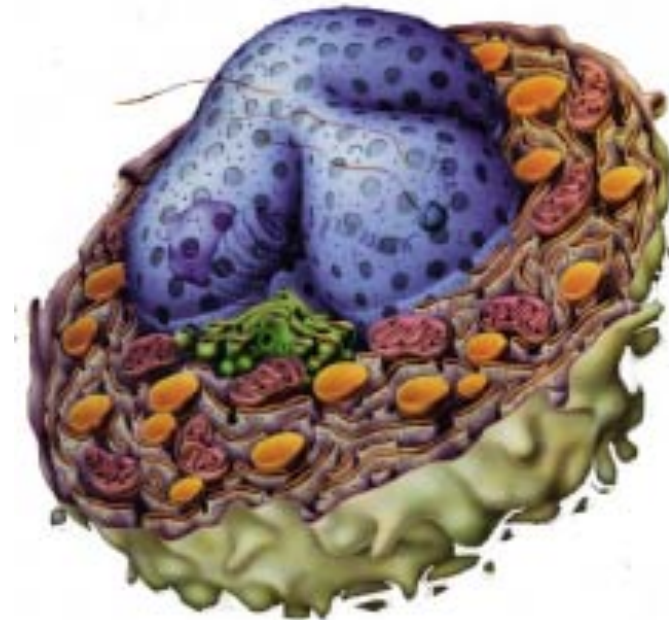


- Biomembrane

400 nm



- Endoplasmic reticulum (ER)

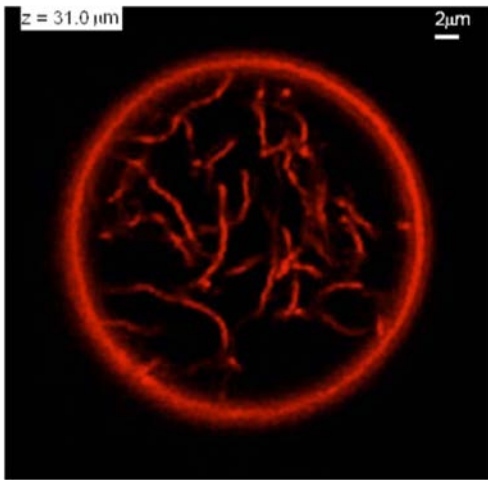


100 μm

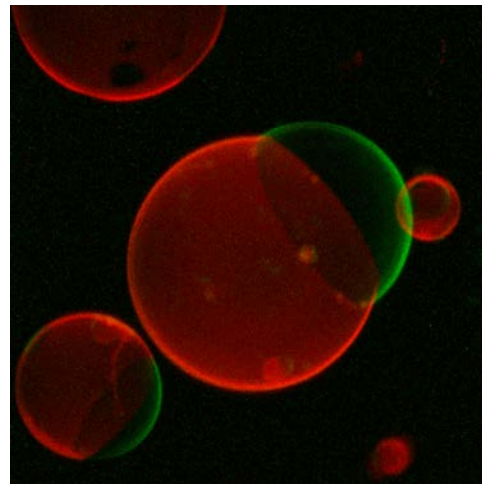
- Animal cell

Morphology of Vesicles

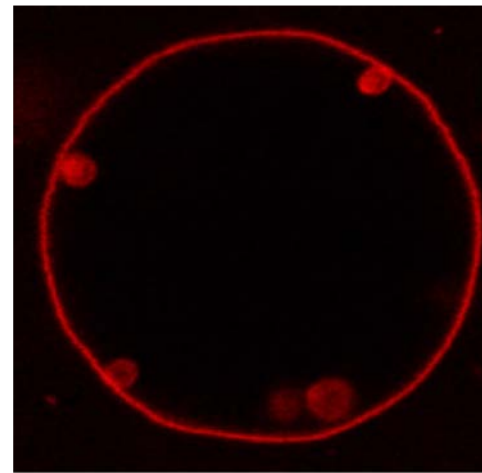
- Giant unilamellar vesicles (GUVs), tens of micrometers
- Remodelling in response to various perturbations:



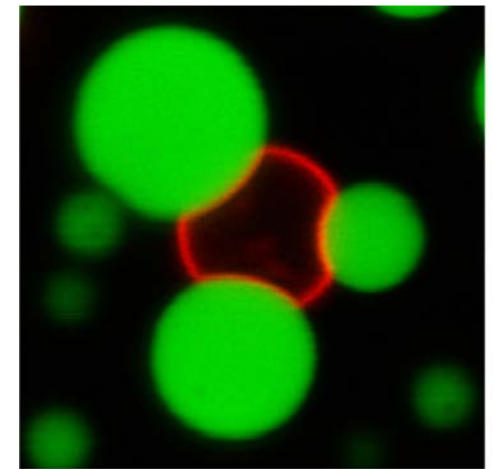
Nanotubes from polymer adsorption, tube width ~ 100 nm



Formation of intra-membrane domains, 2D phase separation



Small buds from protein adsorption, bud size $\sim \mu\text{m}$



Remodelling by adhering or partially wetting droplets

- What determines these strongly nonspherical shapes of membranes and vesicles?

Membrane Fluidity

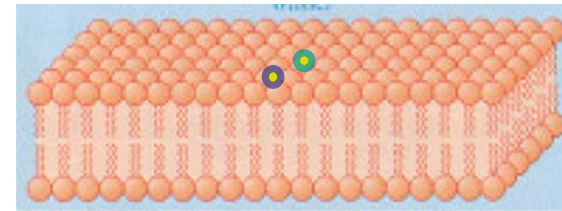
- **Fluid** membranes, i.e., fast lateral diffusion:

Diffusion constant $\sim \mu\text{m}^2/\text{s}$

- Lateral diffusion \Rightarrow demixing and domain formation

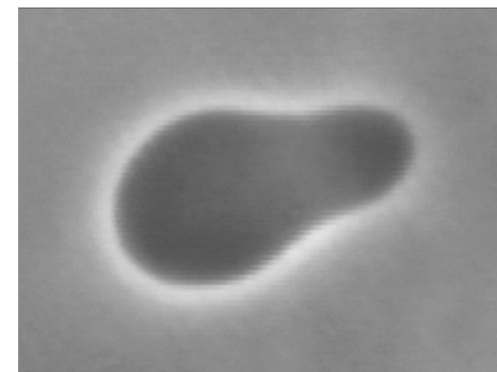
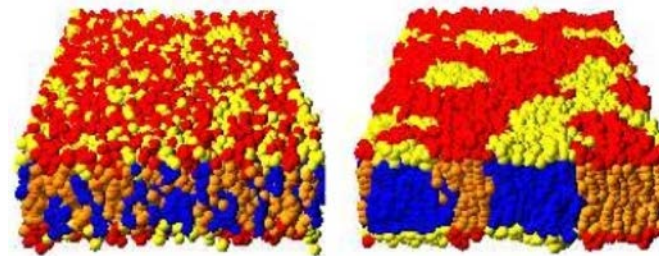
- Fluidity \Rightarrow Flexibility

Formation of spherical bud (with Gaussian curvature) \Rightarrow Direct evidence for fluidity



4 nm

lipid swapping $\sim \text{ns}$



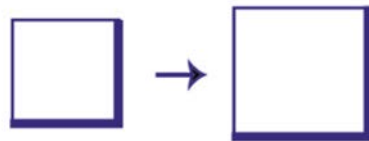
40 μm
5

Membranes as Flexible Sheets

- Membrane as thin elastic sheet

- Elastic Deformations

Stretching



Shearing



Bending



- Fluid Membranes

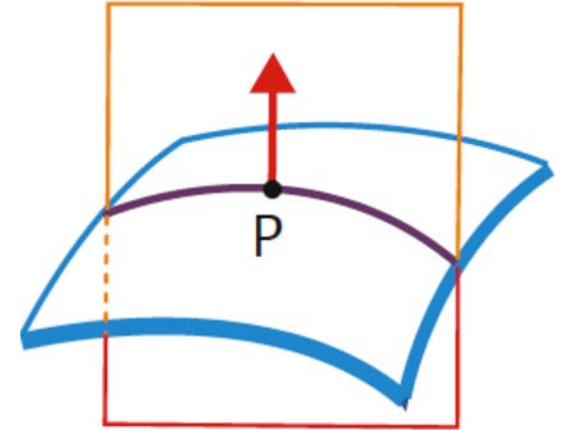
Hardly stretchable
(without rupture)

Shear \rightarrow Flow

Main contribution
related to bending
and curvature

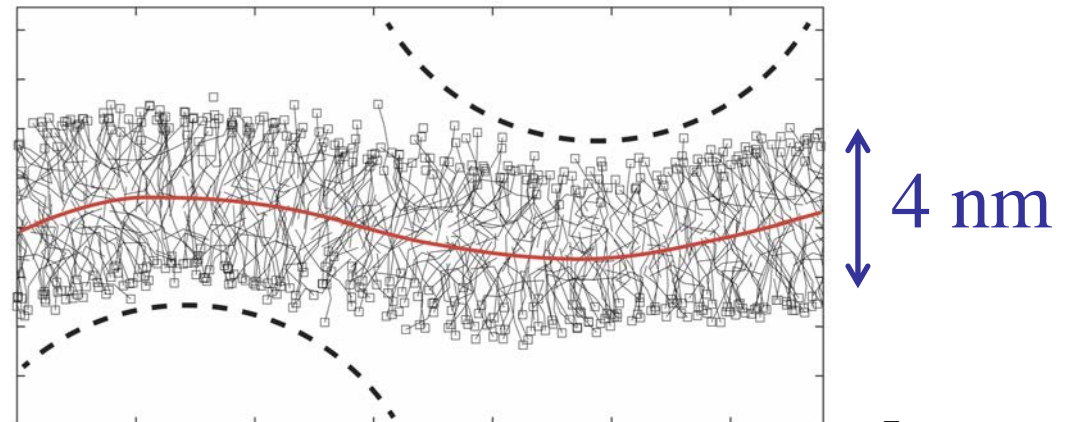
Emergence of Curvature

- Fluid membranes as smooth surfaces:
Fluidity => no intrinsic coordinates
Invariance under coordinate transformations
Principal curvatures C_1 and C_2
Mean curvature $M = (C_1 + C_2)/2$
Gaussian curvature $G = C_1 C_2$



Canham, Helfrich, E. Evans (1970s)

- Mesoscopic description, ignores molecular architecture
- Emergence of curvature on nanoscopic scales:
Bending undulations for membrane patches > 6 nm



Goetz, Gompper, RL (1999)

- Vesicles and Membranes

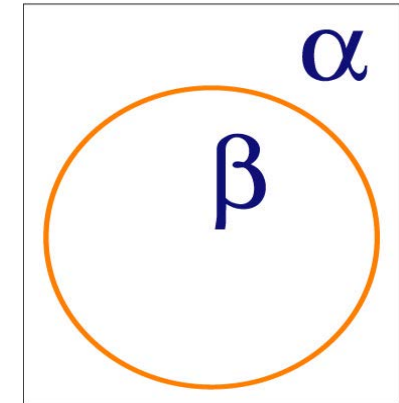


- Membrane tension and curvature elasticity
- Spontaneous curvature and membrane necks
- Morphological complexity of vesicles

For Comparison: Liquid Droplets

- Coexistence of two liquid phases α and β
- Shape energy of a β droplet within the bulk α phase:

$$E = V_{\beta} (P_{\alpha} - P_{\beta}) + A_{\alpha\beta} \Sigma_{\alpha\beta}$$



Two parameters:

Pressure difference $P_{\beta} - P_{\alpha}$ and interfacial tension $\Sigma_{\alpha\beta}$

- First variation of shape energy leads to Laplace equation for mean curvature M :

$$2 M \Sigma_{\alpha\beta} = P_{\beta} - P_{\alpha} = \Delta P$$

- Interfacial tension is constant (for constant temperature)
 \Rightarrow Interface attains shape with **constant M**

Shape Energy of Vesicles

Deuling, Helfrich, *J. Physique* (1976)

- Shape energy of a vesicle

$$E = -\Delta P V + \Sigma A + \text{curvature-elastic terms}$$

- First two terms have the same form as for droplets
- $\Delta P = P_{\text{in}} - P_{\text{ex}}$ is again the pressure difference
- But the area term has a different meaning: in contrast to droplets, the membrane area A is essentially fixed and the tension Σ can either be viewed as a **Lagrange multiplier** to prescribe the area or as the **elastic stress** acting to stretch the weakly compressible membrane

RL, *Adv. Colloid Interface Sci.* (2014)

=> While the interfacial tension $\Sigma_{\alpha\beta}$ represents a material parameter, the membrane tension Σ is shape-dependent

Curvature Elasticity

Canham (1970) Helfrich (1973) Seifert, Berndl, RL (1991)

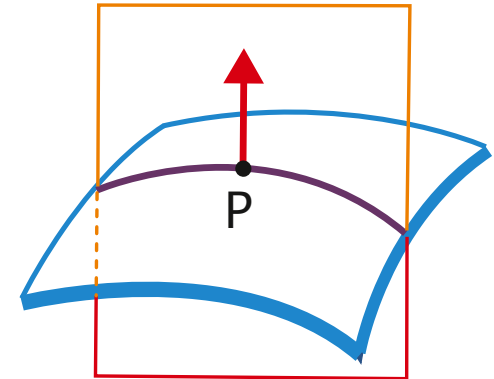
- Local mean curvature M tries to adapt to spontaneous (or preferred) curvature m
- Curvature or bending energy:


$$E_{cu} = \int dA 2 \kappa (M - m)^2$$

integral over membrane area A

- 2nd fluid-elastic parameter: Bending rigidity κ
Dimensions of energy, $\kappa = 10^{-19} \text{ J} = 20 k_B T$
- Range of spontaneous curvatures m

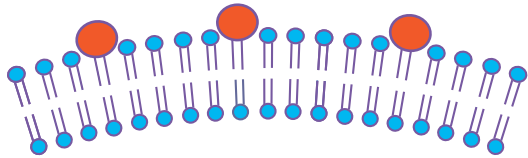
from $1/(20 \text{ nm})$ to $1/(20 \mu\text{m})$



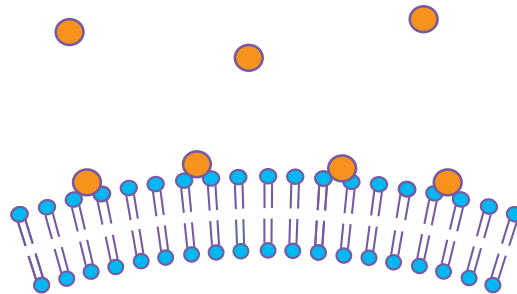
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Spontaneous or Preferred Curvature

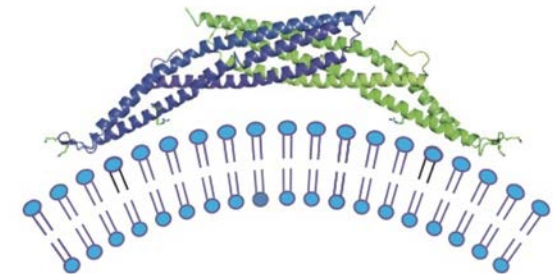
- Spontaneous or preferred curvature m describes bilayer asymmetry = asymmetry between two leaflets
- Different molecular mechanisms for bilayer asymmetry:



Asymmetric
composition,
e.g., ganglioside



Asymmetric
adsorption of
small molecules

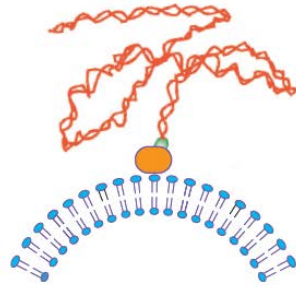


Asymmetric
protein coats,
e.g. BAR-domain

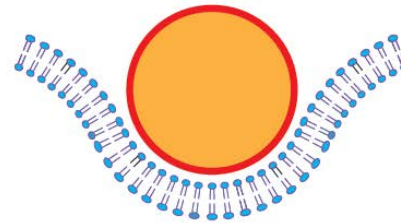
Local Curvature Generation

- Local curvature generated on nanoscopic scales:

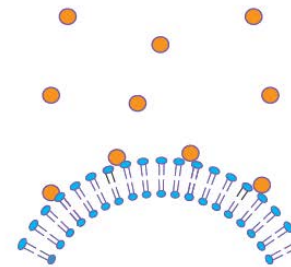
Anchored polymer



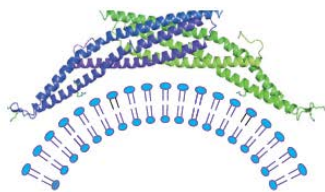
'Large' particle



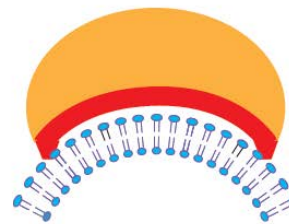
'Small' adsorbate particles



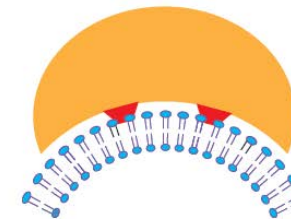
BAR-domain protein



Nonspherical Janus particles



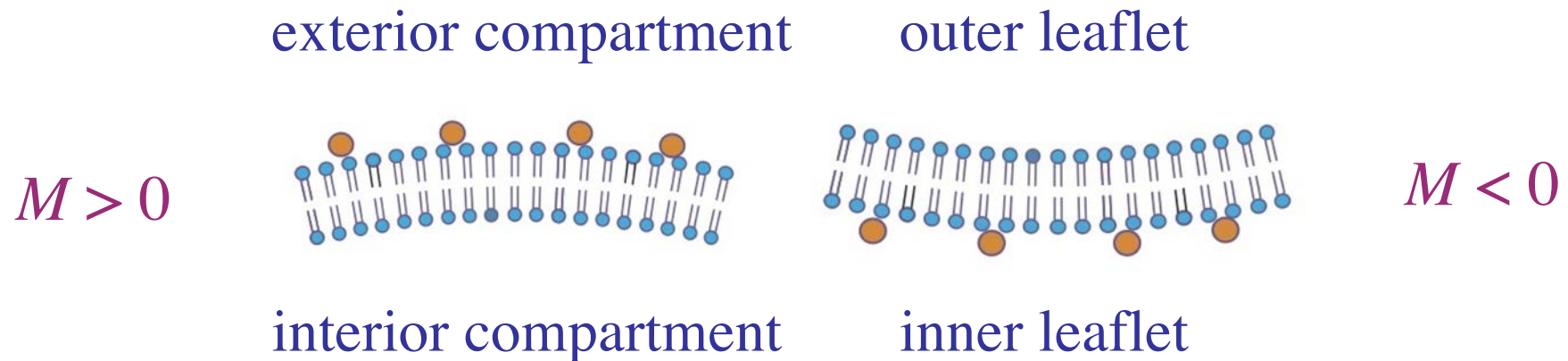
Induced Fit



Conformational Selection

Sign of (Spontaneous) Curvature

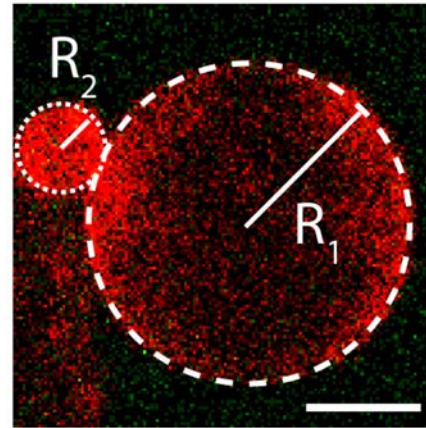
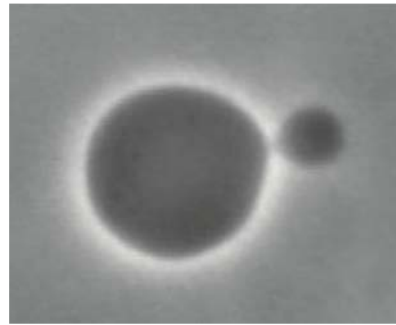
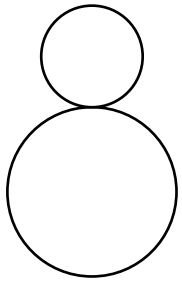
- Mean curvature M and spontaneous curvature m can be positive or negative
- Sign defined with respect to interior/exterior compartments = with respect to inner/outer leaflet



Mean curvature M is positive (negative) if membrane bulges towards exterior (interior) compartment

(1+1)-Spheres and Membrane Necks

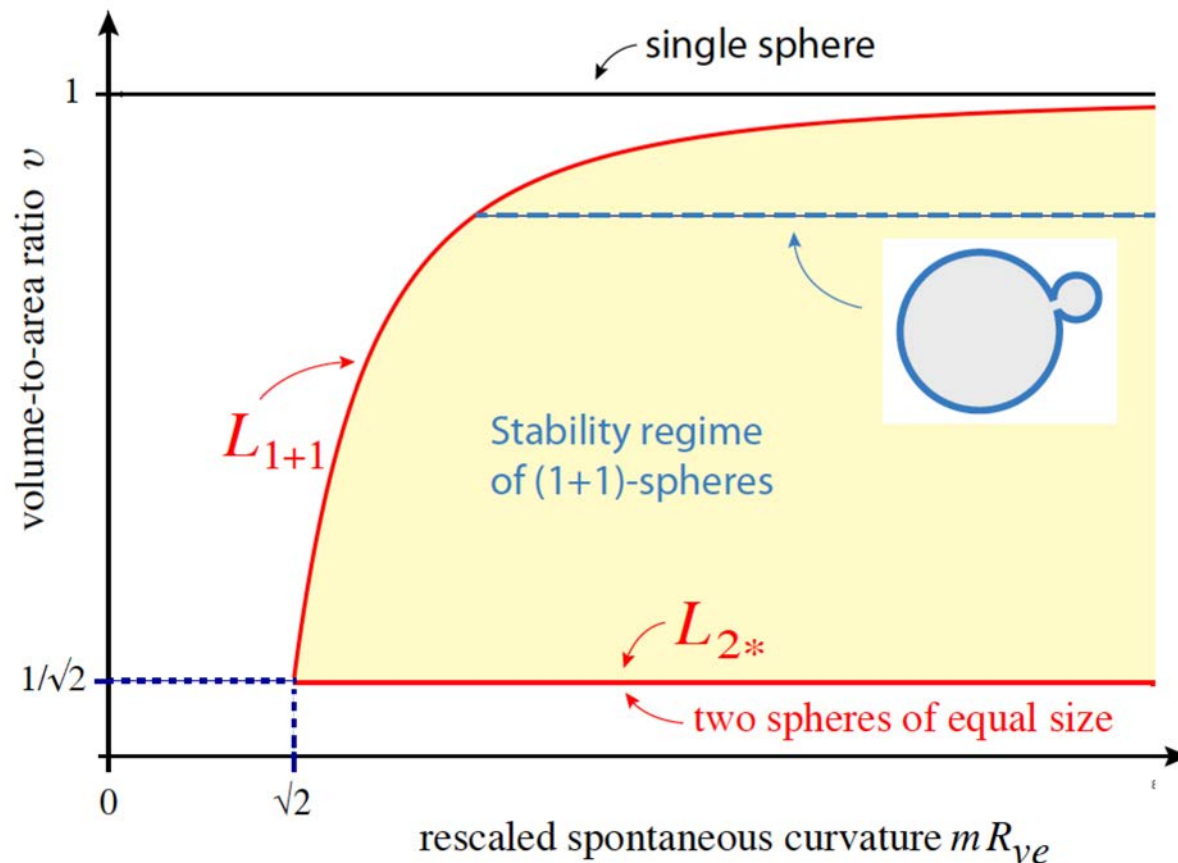
- Positive sp-curvature $m > 0$ leads to spherical membrane segments connected by closed membrane necks :



- Sphere radii R_1 and R_2
- Mean curvatures $M_1 = 1/R_1$ and $M_2 = 1/R_2$
- Neck curvature $M_{\text{ne}} = (1/2) (M_1 + M_2)$
- Closed neck is stable if $0 < M_{\text{ne}} \leq m$
- **Local** relation between geometry and material parameter

Stability Regime of (1+1)-Spheres

- Vesicle area A , length scale = vesicle size $R_{ve} = [A/(4\pi)]^{1/2}$
- Dimensionless volume $v \sim V/R_{ve}^3$ with $0 < v \leq 1$
- Dimensionless sp-curvature $mR_{ve} > 0$



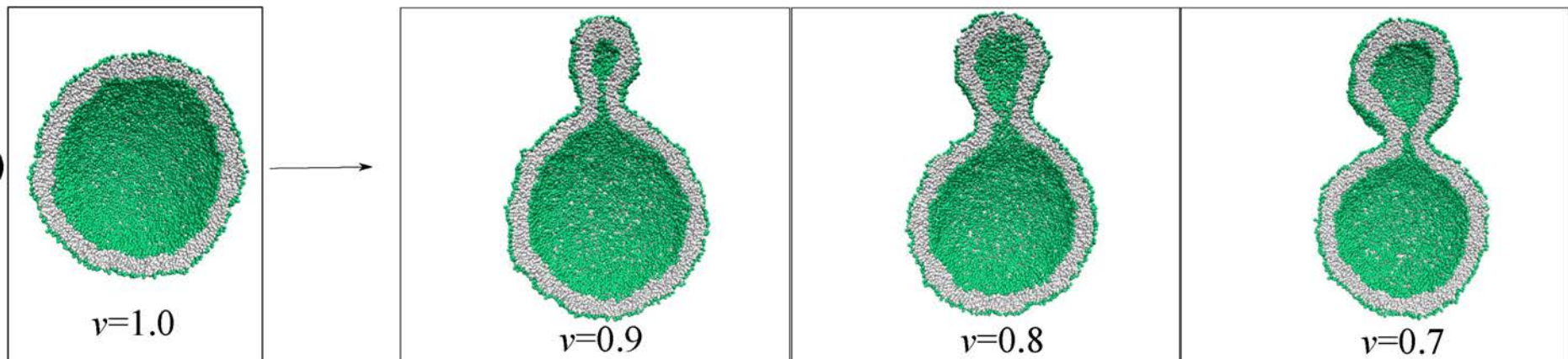
within yellow
stability regime:
dumbbell shape
depends only
on v and not on mR_{ve}

volume v :
 $1 > v \geq 0.707$

Membrane Necks of Nanovesicles

Rikhia Ghosh, Vahid Satarifard, A. Grafmüller, RL. *Nano Letters* (2019)

- Molecular dynamics simulations
- Spherical nanovesicle with diameter of 40 nm
- Decreasing vesicle volume v , corresponding to deflation
- Formation of dumbbell with closed membrane neck:



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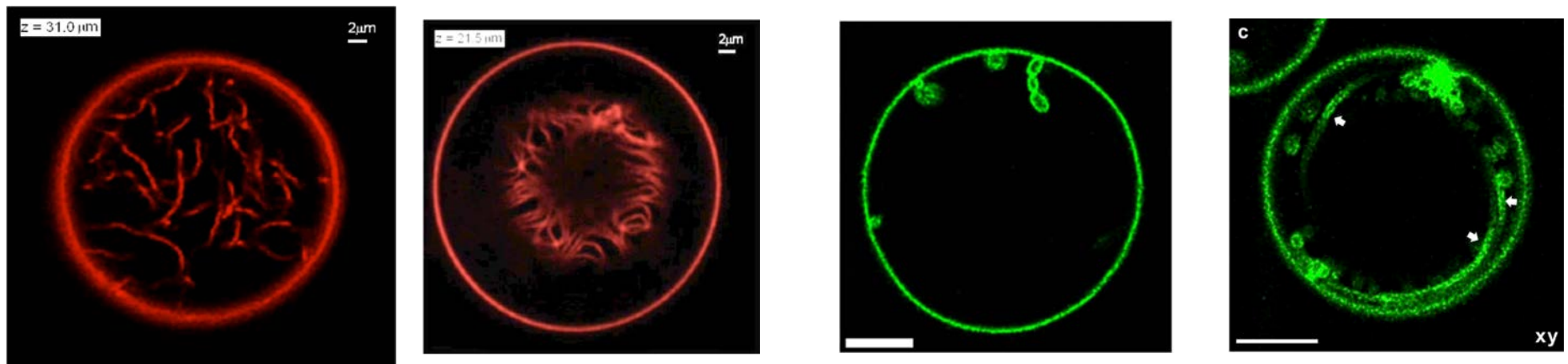


Large *sp*-curvature: Membrane nanotubes
Controlled *sp*-curvature and GUV division

Large Sp-Curvature: Nanotubes

Li ... Dimova, *PNAS* (2011) Liu ... RL, *ACS Nano* (2016)

- Lipid mixture of DOPC, DPPC, cholesterol
- Small amounts of fluorescently labeled lipids
- Liquid-disordered (red) and liquid-ordered phase (green)



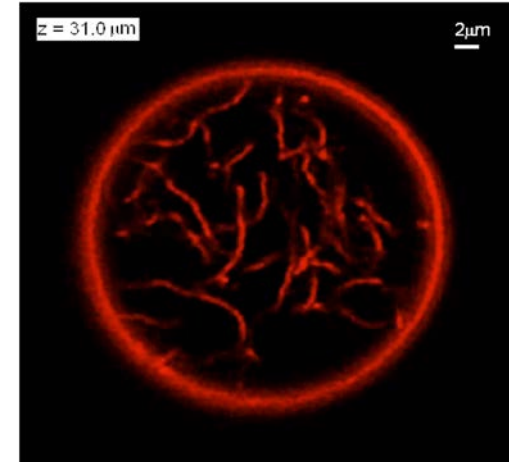
- Spontaneous tube formation **without** external forces
- Complex patterns of nanotubes

Sp-Tubulation and Sp-Tension

RL, *Faraday Discuss.* (2013)

- Tubulation leads to tense mother vesicle
- Total tension in Euler-Lagrange equation has two components:

$$\hat{\Sigma} = \Sigma + \sigma$$



Mechanical tension Σ stretches the membrane

Spontaneous tension $\sigma = 2 \kappa m^2$ for $M \ll m$

- Presence of nanotubes implies dominance of spontaneous tension, mechanical tension can be ignored
- Example: Spont curvature $\approx -1/(100 \text{ nm})$

Spontaneous tension $\sigma \approx 10^{-2} \text{ mN/m}$

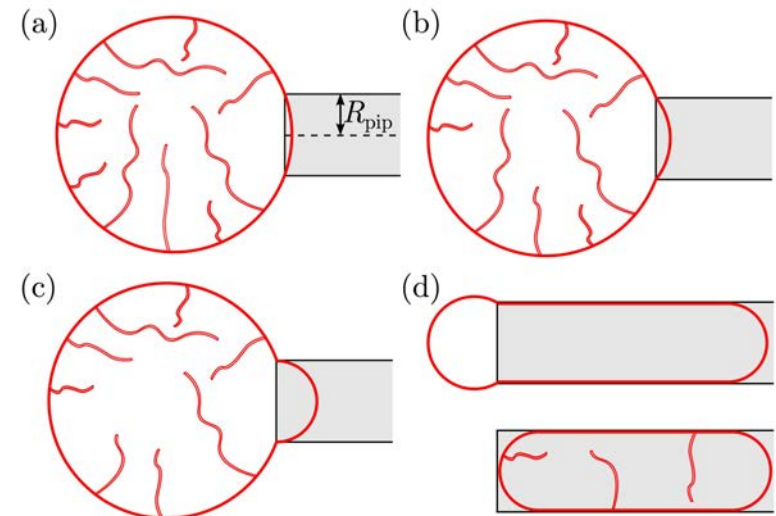
Mechanical tension $\Sigma \approx 10^{-4} \text{ mN/m}$

Robustness of tubulated GUVs

- Conventional GUVs: Membrane rupture under osmotic inflation, strong adhesion, micropipette aspiration, ...
- Membrane nanotubes provide area reservoir
- Tubulated GUVs have very low mechanical tension and do **not** rupture under strong mechanical perturbations

Bhatia ... RL: *ACS Nano* (2018)

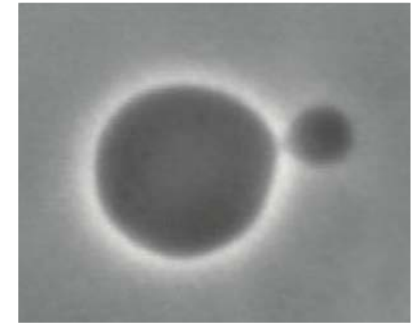
- Robustness demonstrated for inflation and aspiration
- Membrane tension dominated by sp-tension $\sigma = 2 \kappa m^2$
- Mother vesicle behaves like liquid droplet with interfacial tension = sp-tension of membrane



How Do Nanotubes Form?

Liu et al, *ACS Nano* (2016)

- Tubulation intimately related to budding
- Osmotic deflation of spherical GUV
- Small deflation step leads to single bud
- Bud and mother vesicle connected by membrane neck
- Bud acts as nucleation site for necklace-like tube
- Several pathways for subsequent deflation steps:
 - Formation of new bud
 - Bud into 2-necklace
 - N-necklace into (N+1)-necklace

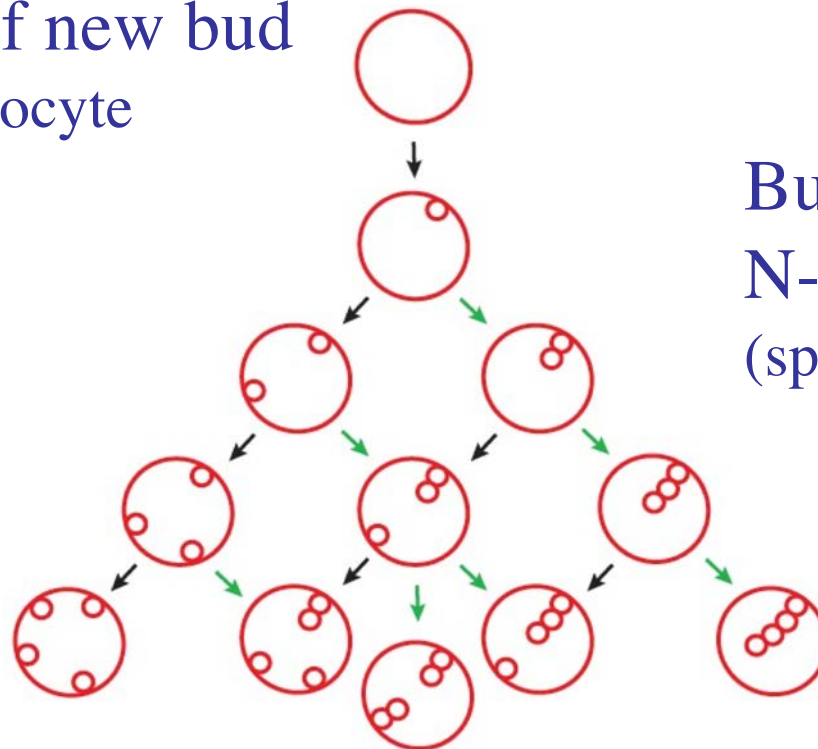


Nucleation and Growth of Tubes

Liu et al, *ACS Nano* (2016)
RL, *J. Phys. D* (2018)

- Spherical GUV, large spont curv m
- Osmotic deflation of GUV in discrete steps
- At each step, different morphological pathways:

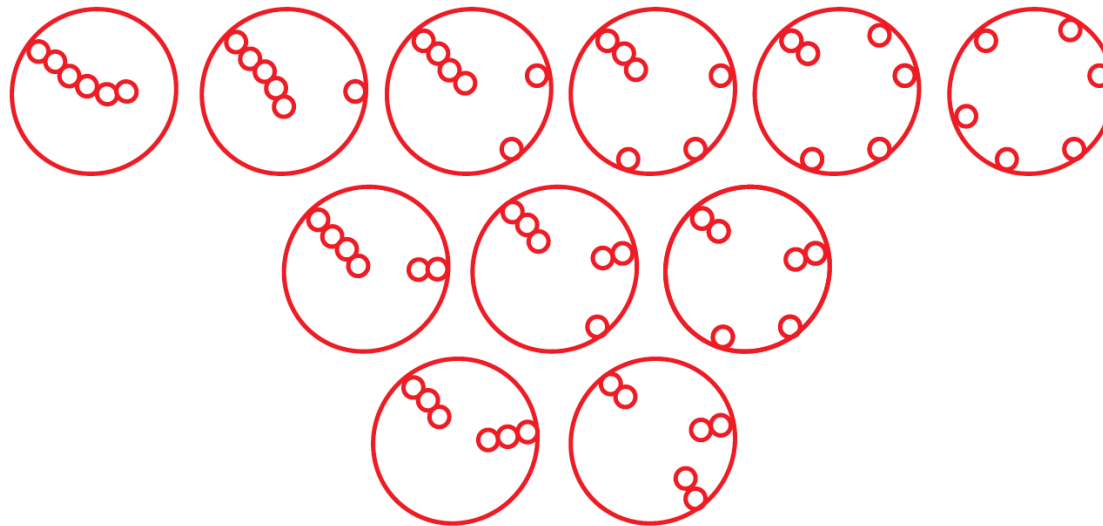
Formation of new bud
(oblate-stomatocyte
bifurcation)



Bud into 2-necklaced or
N- into (N+1)-necklace
(sphere-prolate bifurcation)

Morph Complexity: Theory

- After 6th step, 11 morphologies with 6 spherules:

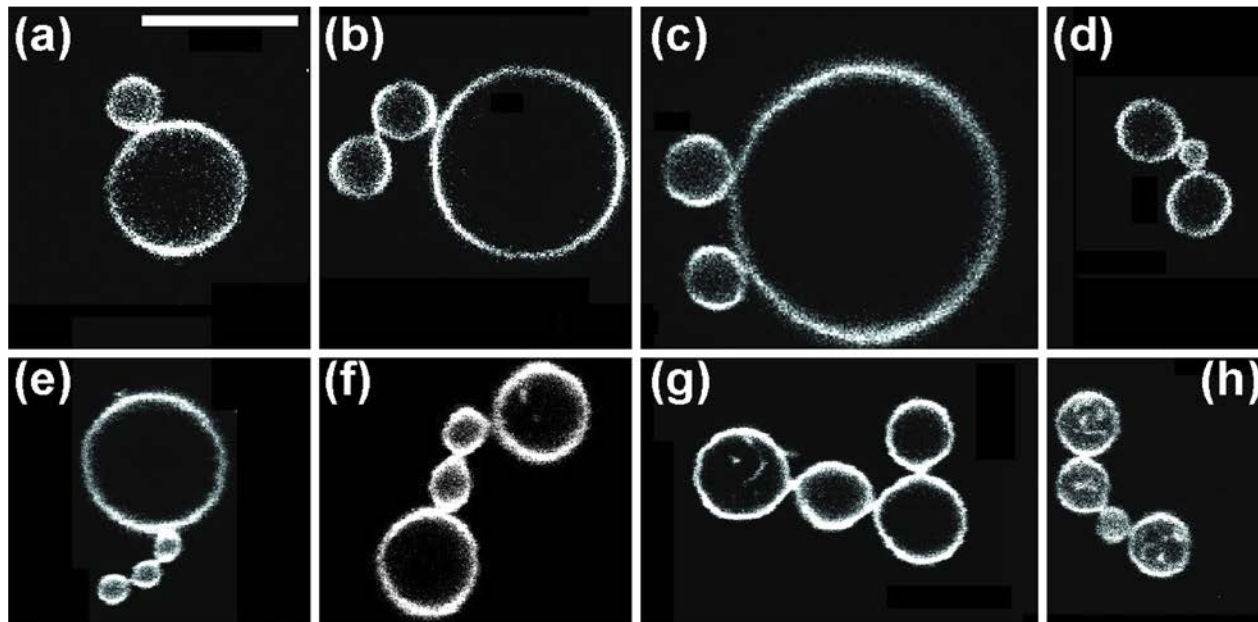


- All beads are connected by membrane necks
- All morphologies have the same area, volume, and curvature energy
- Rugged energy landscape contains 11 intersecting branches
- For large N , number of N -spherule morph grows as $\exp[c \sqrt{N}]$

Morph Complexity: Experiment

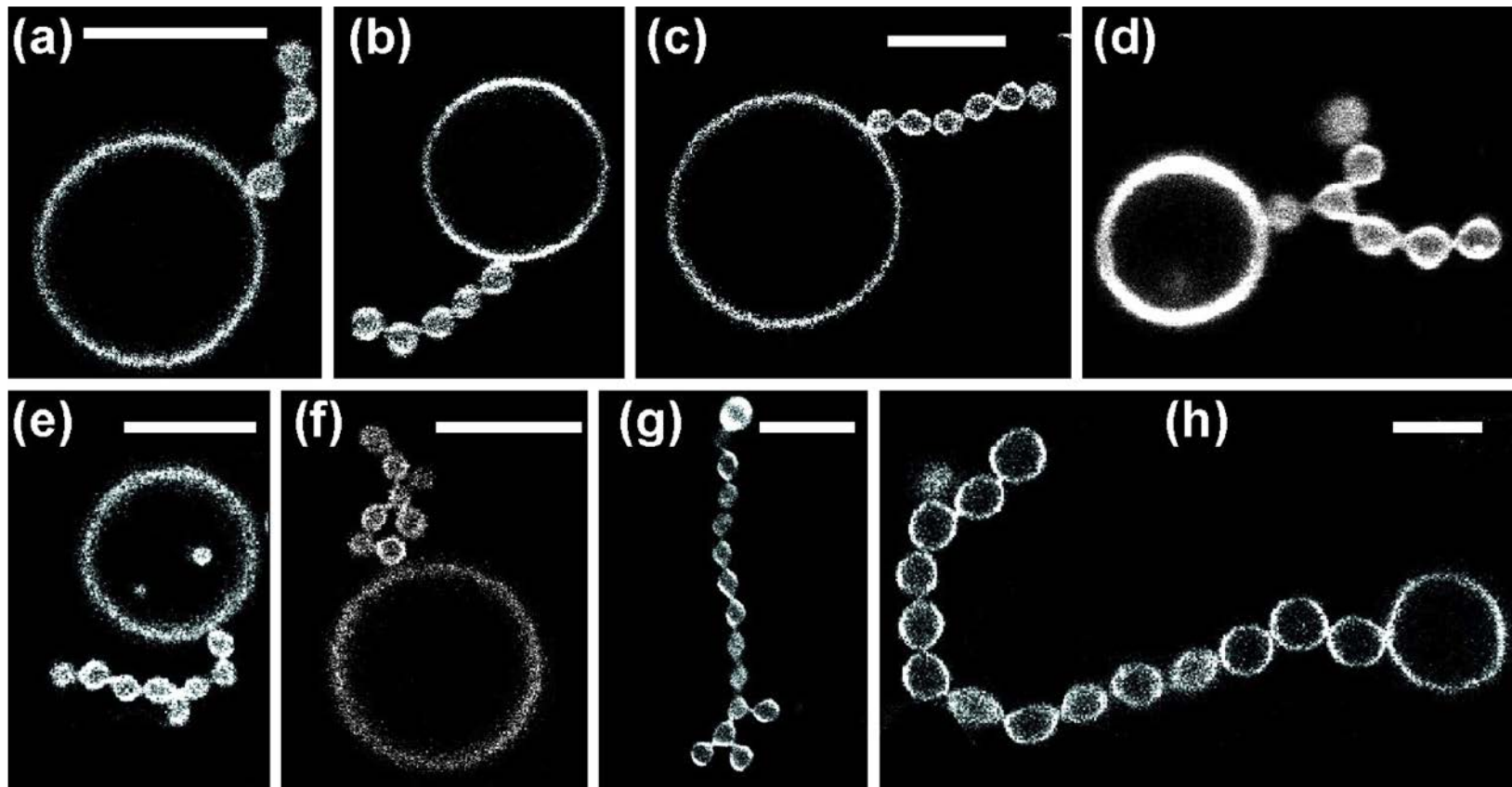
Bhatia, Christ, Steinkühler, Dimova, RL, *Soft Matter* (under revision)

- GUVs exposed to two simple sugars, sucrose and glucose
- Sugar asymmetry: sucrose inside, glucose outside
- Single GUV membrane forms N_l large and N_s small spheres connected by closed membrane necks
- Multispheres with $N = N_l + N_s$ and $2 \leq N \leq 4$:



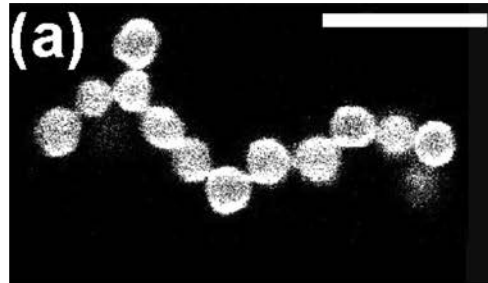
Multispheres with $N = 1 + N_s$

- One large and N_s small spheres, linear and branched chains:

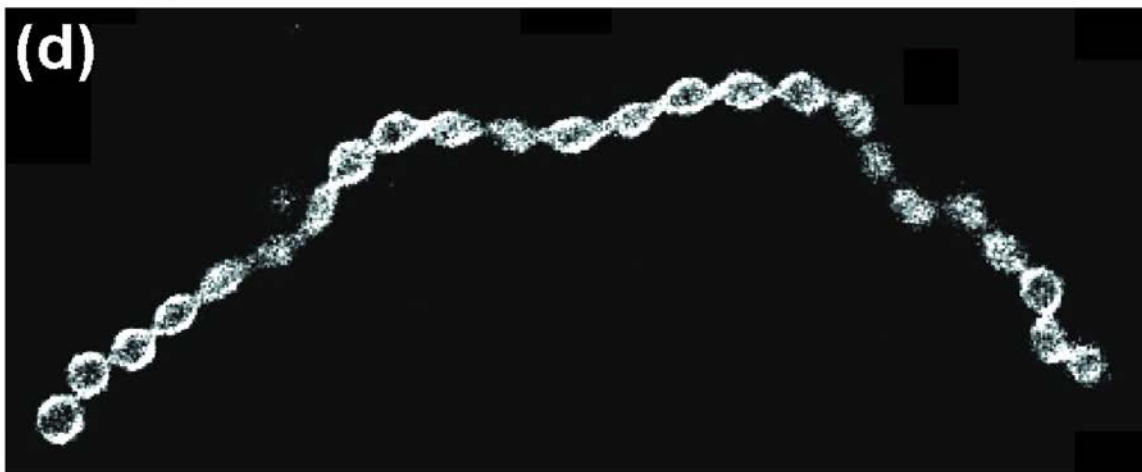
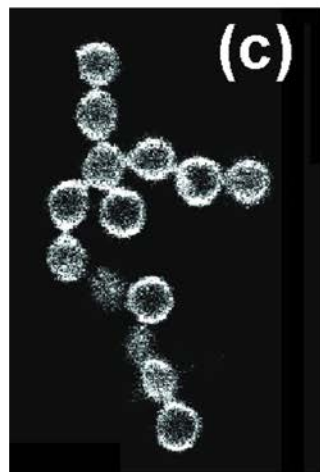


- Individual small spheres are surprisingly mobile !

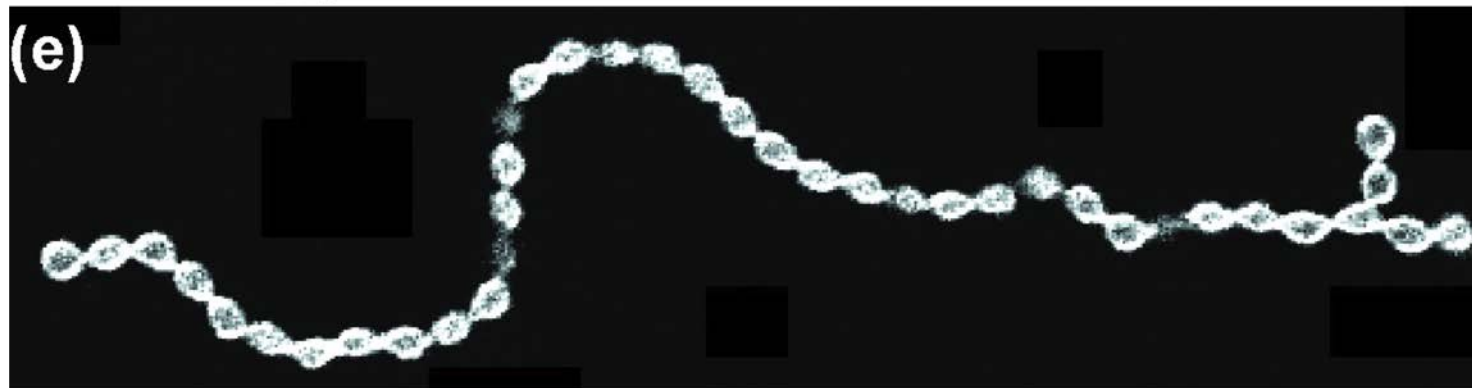
N_* Equally Sized Spheres



$N_* = 15$



$N_* = 24$



$N_* = 39$

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- Spontaneous curvature and membrane necks
- Morphological complexity of vesicles

Large sp-curvature: Membrane nanotubes

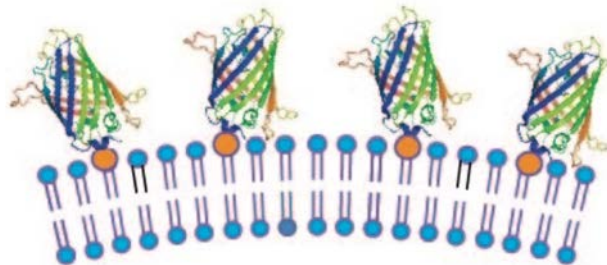
Controlled sp-curvature and GUV division



Controlled Variation of Sp-Curvature

Steinkühler, ... , Wegner, Dimova, RL, *Nature Comm* (under revision)

- Binding of GFP to certain anchor lipids:

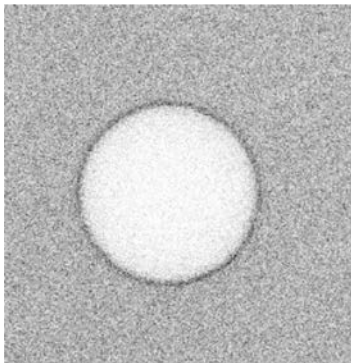


His-tagged GFP
NTA-lipids

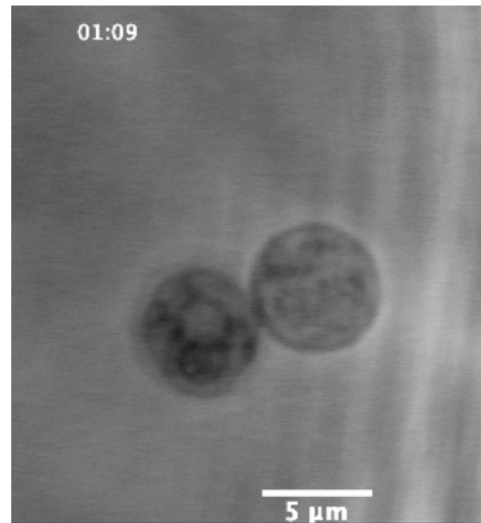
- GFP solution concentration X as control parameter
- Density Γ of bound GFP increases linearly with X
- Sp-curvature m increases linearly with Γ
- Dilute regime: separation of bound GFPs
much larger than lateral size of GFP

Controlled Division of GUVs

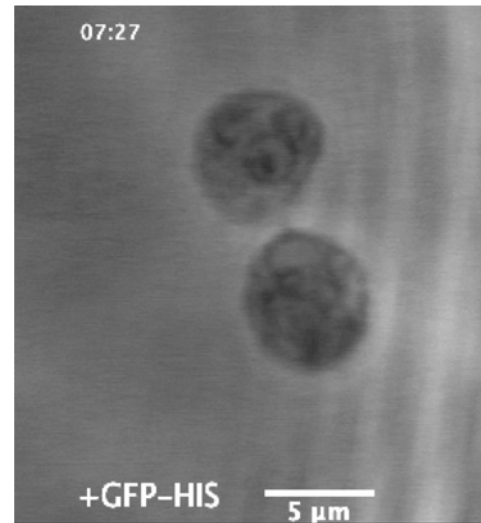
- Osmotic deflation + GFP binding
- Osmotic deflation: Spherical GUV \rightarrow dumbbell GUV
Increase in GFP \rightarrow Neck cleavage \rightarrow Two daughter GUVs



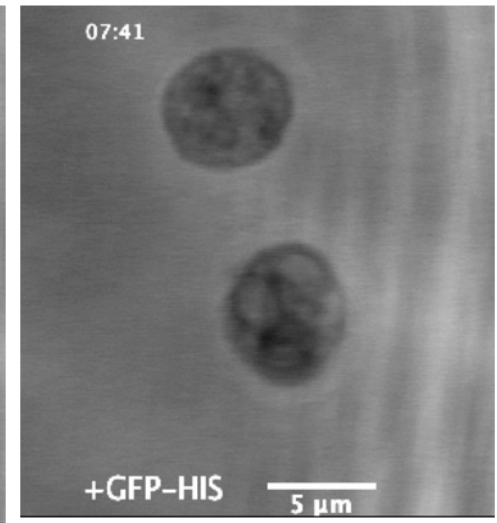
Adsorption of GFP onto GUV membrane



Deflation leads to dumbbell with membrane neck



Directly after neck cleavage



Complete division into two smaller GUVs

Constriction Force from Sp-Curvature

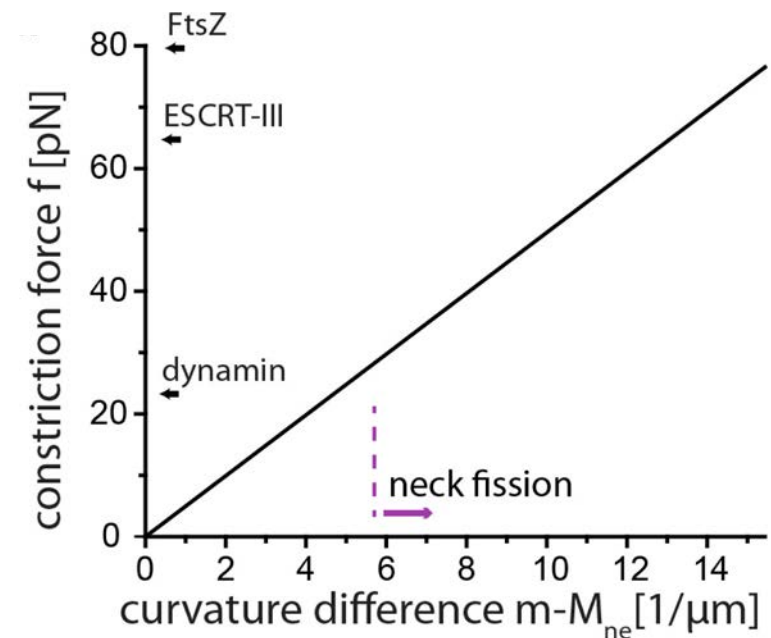
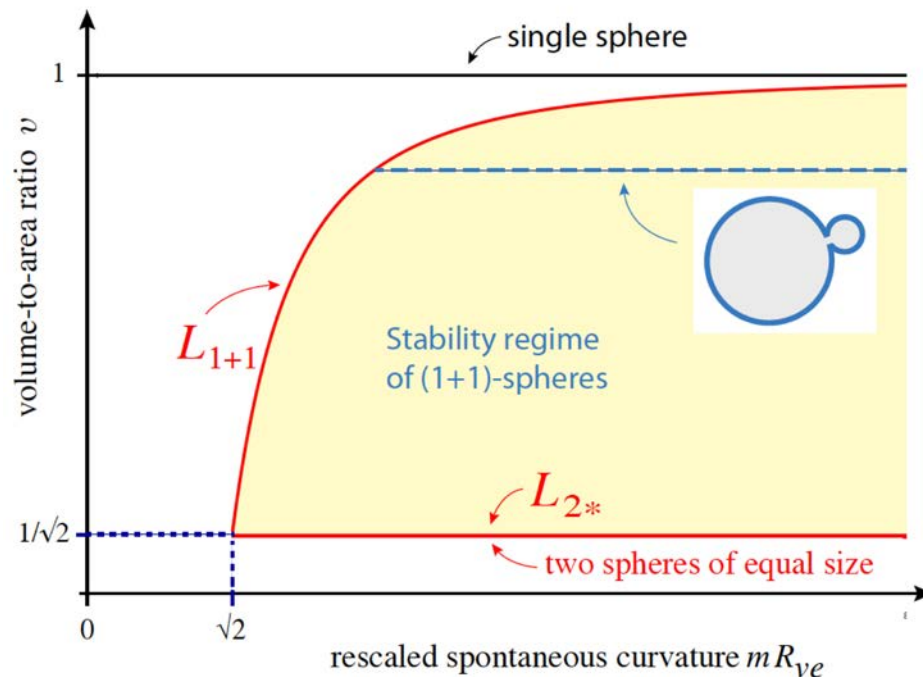
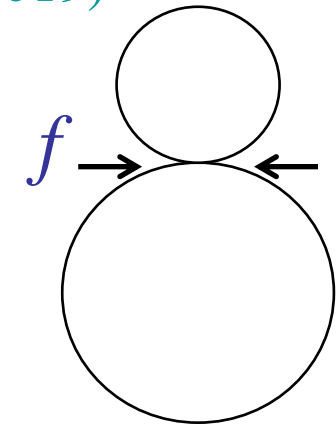
RL, in *The Giant Vesicle Book*. ed by Dimova and Marques (2019)

- Sp-curvature generates constriction force

$$f = 8\pi \kappa (m - M_{ne})$$

acting radially on closed membrane neck:

- Force increases with increasing sp-curvature:



- Vesicles and Membranes
- Membrane tension and curvature elasticity
- Spontaneous curvature
- Morphological complexity of vesicles
- Outlook on related topics:



Bilayer membranes and leaflet tensions

Endocytosis of nanoparticles by membranes

Wetting and engulfment of droplets

...

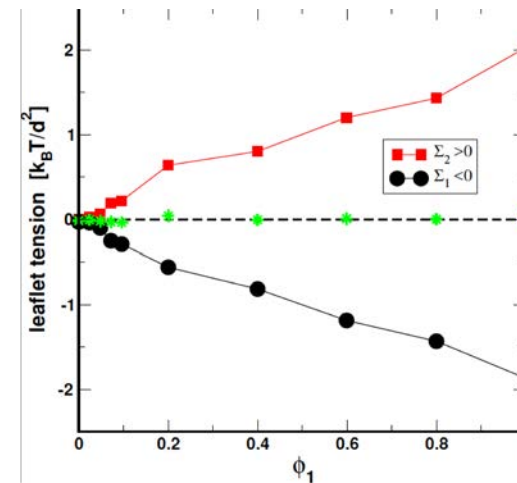
Membrane Tension

- Uniform membranes: Laterally uniform composition and spatially uniform environment
- Uniform membranes experience mechanical tension Σ and spontaneous tension $\sigma = 2 \kappa m^2$
- These two tensions act on the whole membrane
- Optimal packing of lipids \Leftrightarrow low mech tension Σ
 \Leftrightarrow tensionless membrane
- **But** biomembranes are molecular bilayers
- What about tensions within individual leaflets?

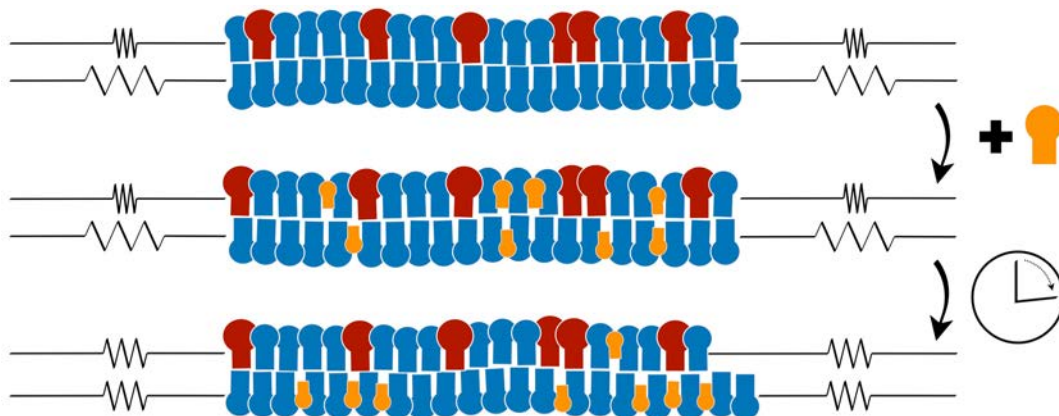
Bilayer Membranes and Leaflet Tensions

- Bilayer with two leaflets:
 - Two leaflet tensions Σ_1 and Σ_2 with $\Sigma_1 + \Sigma_2 = \Sigma$
 - Tensionless bilayer: $\Sigma = 0$
 - Leaflet tensions for binary mixture

Sreekumari, RL , J. Chem. Phys. (2018)



- Leaflet tensions and flip-flops:



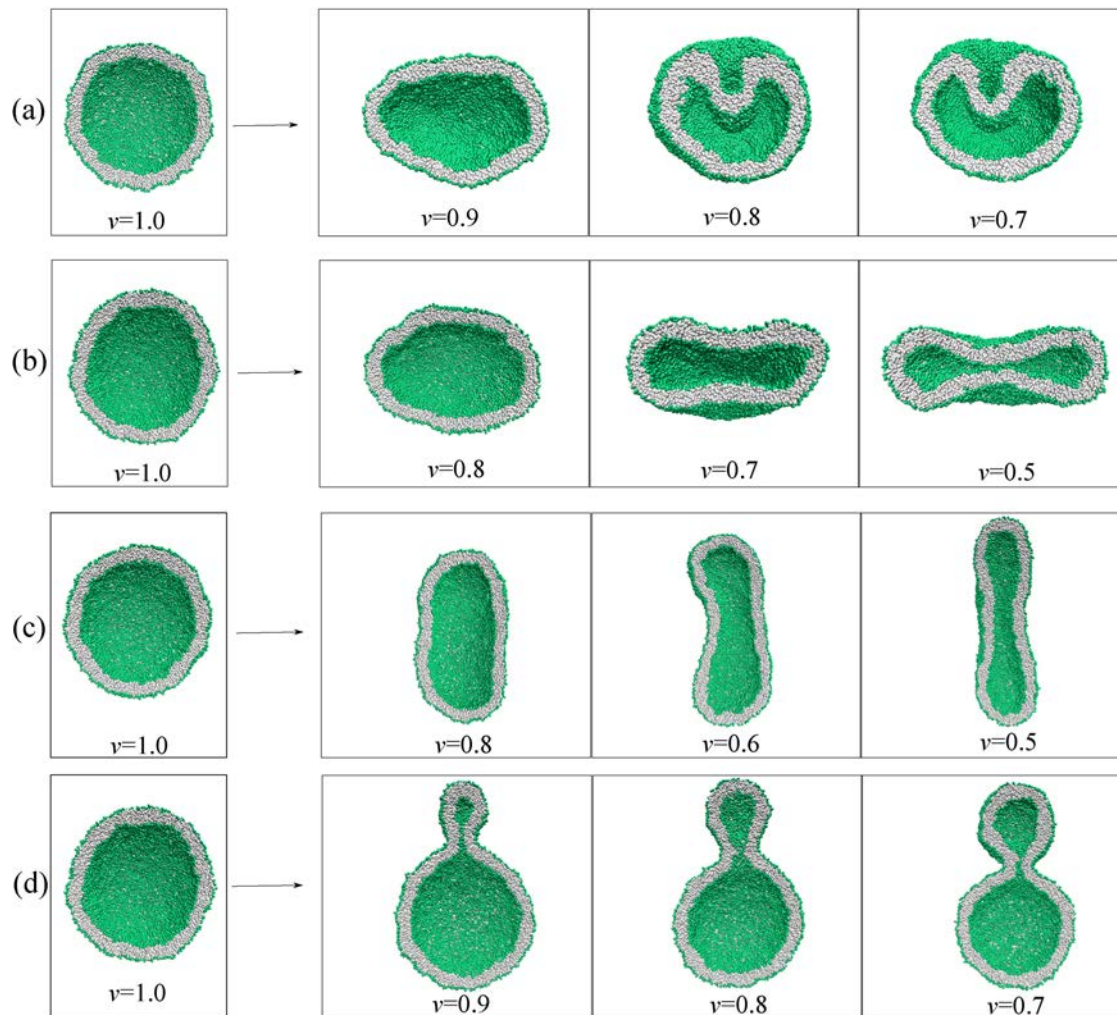
Miettinen, RL , Nanoletters (2019)

- Add cholesterol
- Leaflet tensions relax towards $\Sigma_1 = \Sigma_2 = 0$

Leaflet Tensions and Nanovesicles

Ghosh, Satarifard, Grafmüller, RL, *Nanoletters* (2019)

- Polymorphism of spherical nanovesicle:



- Four spherical vesicles
- Same volume
- Same total # of lipids
- Reduction of volume: very different shapes
- Shape transformations determined by leaflet tensions Σ_1 and Σ_2

- Vesicles and Membranes
- Membrane tension and curvature elasticity
- Spontaneous curvature
- Morphological complexity of vesicles
- Outlook on related topics:

Bilayer membranes and leaflet tensions



Endocytosis of nanoparticles by membranes

Wetting and engulfment of droplets

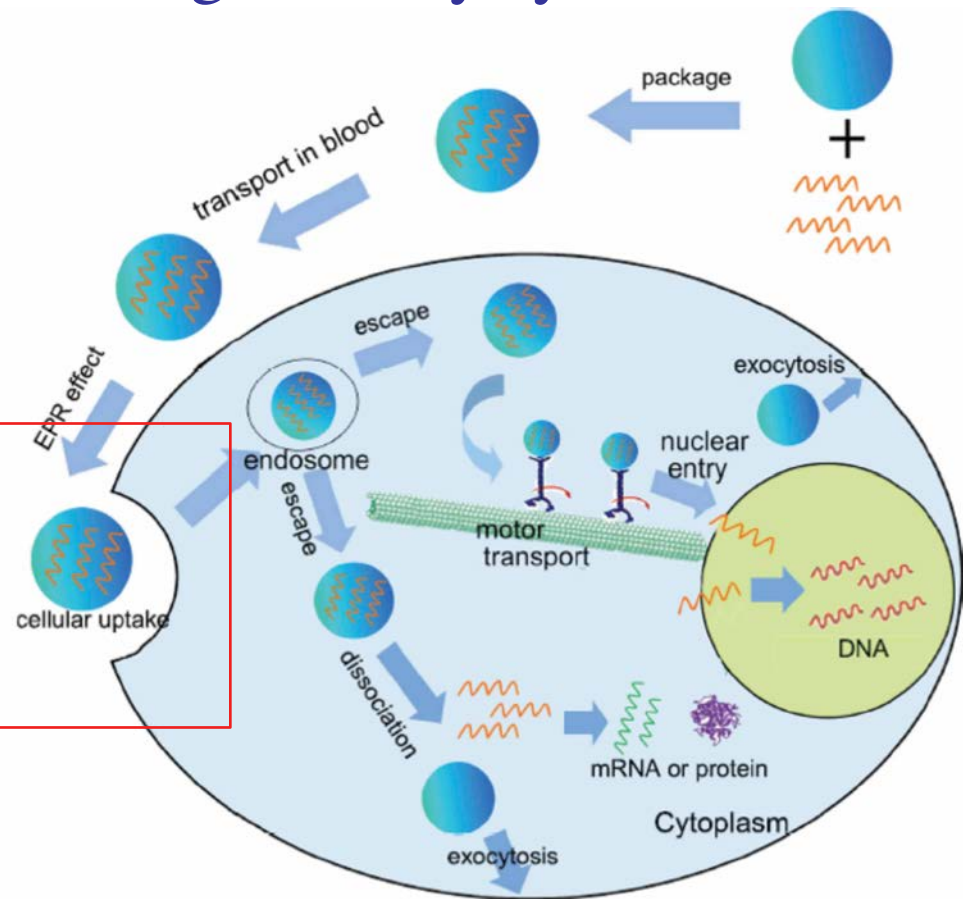
...

Targeting Nanoparticles to Cells

- Nanoparticles (NPs) as drug delivery systems:

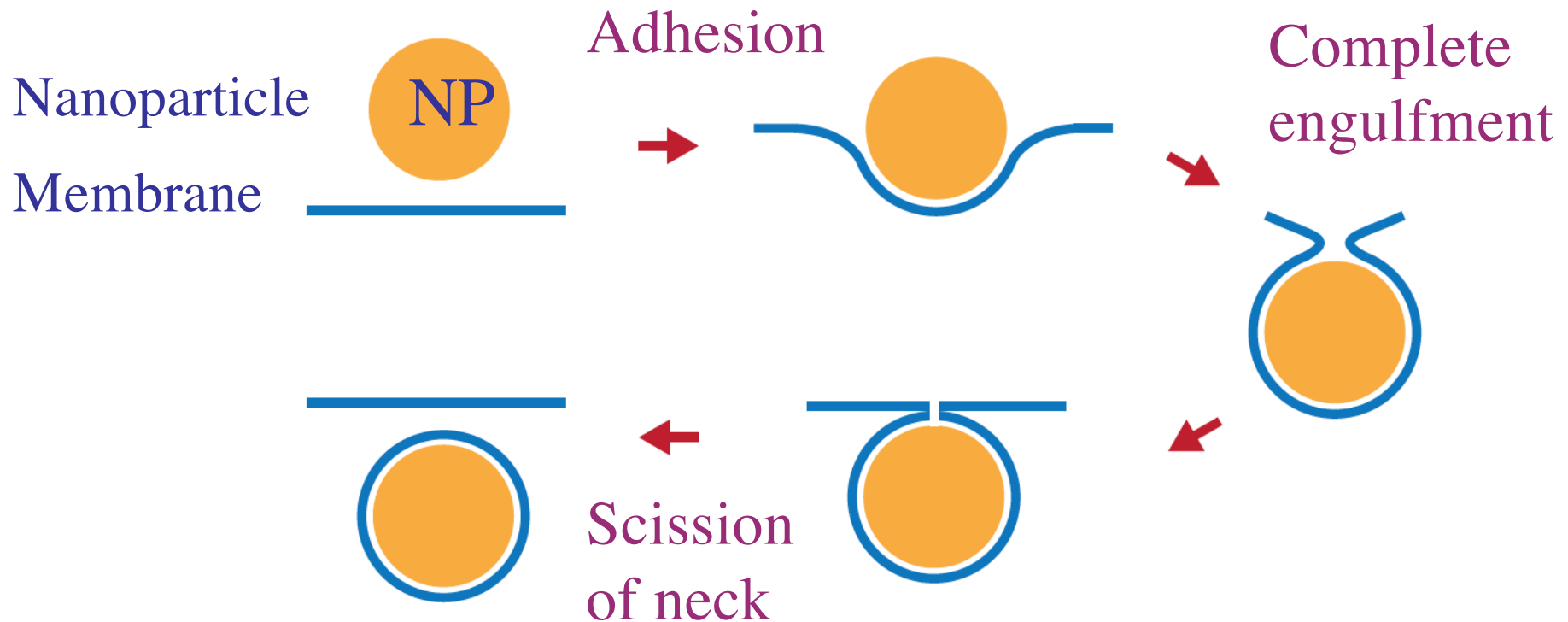
Transport of NPs towards cells

Transport across cell membrane by endocytosis



- Endocytic pathway also used by viruses, airborne ultrafine particles, ...

Endocytosis of Nanoparticles



- Three steps: Adhesion, Complete Engulfment, Scission
- All steps governed by **local** stability relations

Adhesion: Basic Aspects



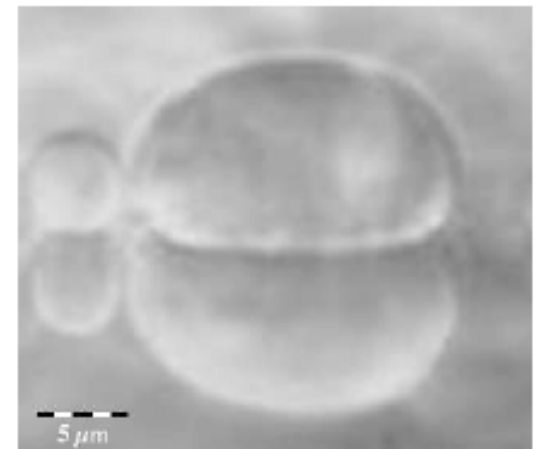
- Attractive interactions between NP and membrane
- Van der Waals, electrostatic, receptor-ligand
- Gain of adhesion free energy but increase of elastic membrane energy
- Competition between adhesion and bending
- Bending rigidity κ versus adhesive strength $|W|$

Adhesion Length

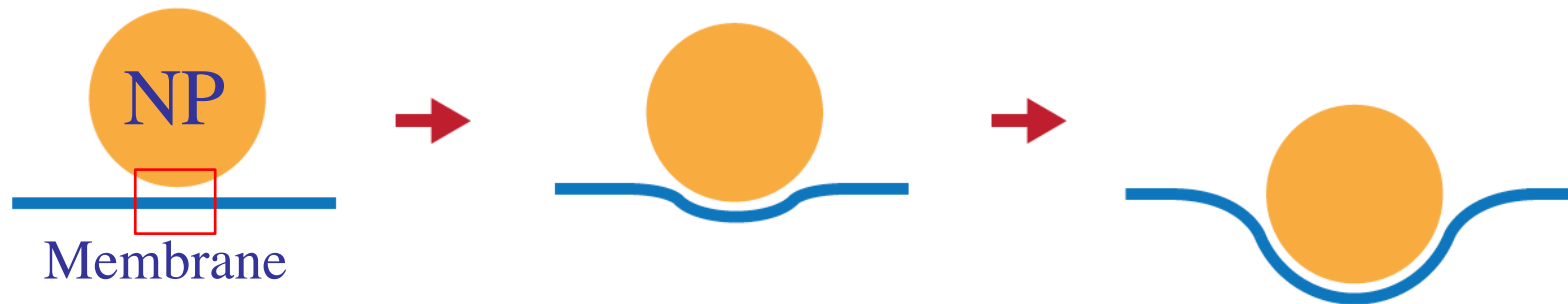
- Adhesive strength $|W|$ = adhesion free energy per area
- Bending rigidity κ and adhesive strength $|W|$ define adhesion length

$$R_W = (2\kappa/|W|)^{1/2}$$

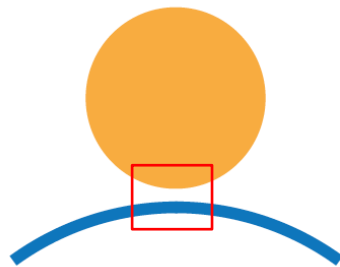
- For specific NP-membrane systems, R_W varies between 10 nm and 3 μm !
- Large R_W values can be measured via membrane curvature along contact line



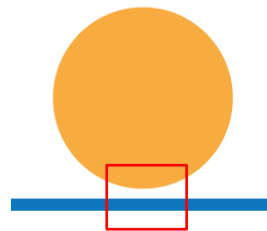
Onset of Adhesion: Key Parameters



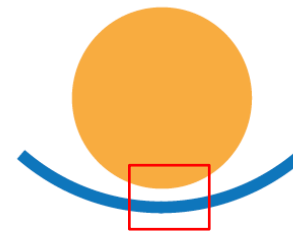
- Three key parameters for onset of adhesion:
Adhesion length R_W , Particle size R_{pa} , and
Membrane curvature M at point of contact
- Membrane curvature M can be positive or negative:



$$M > 0$$



$$M = 0$$



$$M < 0$$

Onset of Adhesion: Local Criterion

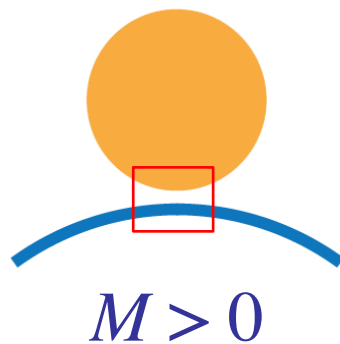
Agudo-Canalejo and RL, *ACS Nano + Nano Letters* (2015)

- Membrane starts to spread over particle if

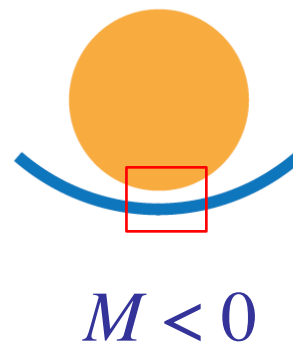
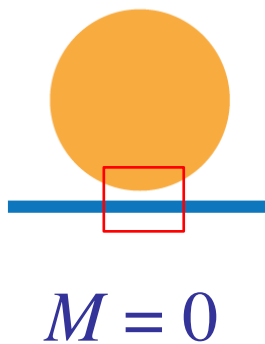
$$M \leq 1/R_W - 1/R_{pa} =: M_{co}$$

contact curvature
 M_{co} is threshold
value for M

- Example: $R_W = R_{pa}$ or $M_{co} = 0$



no adhesion

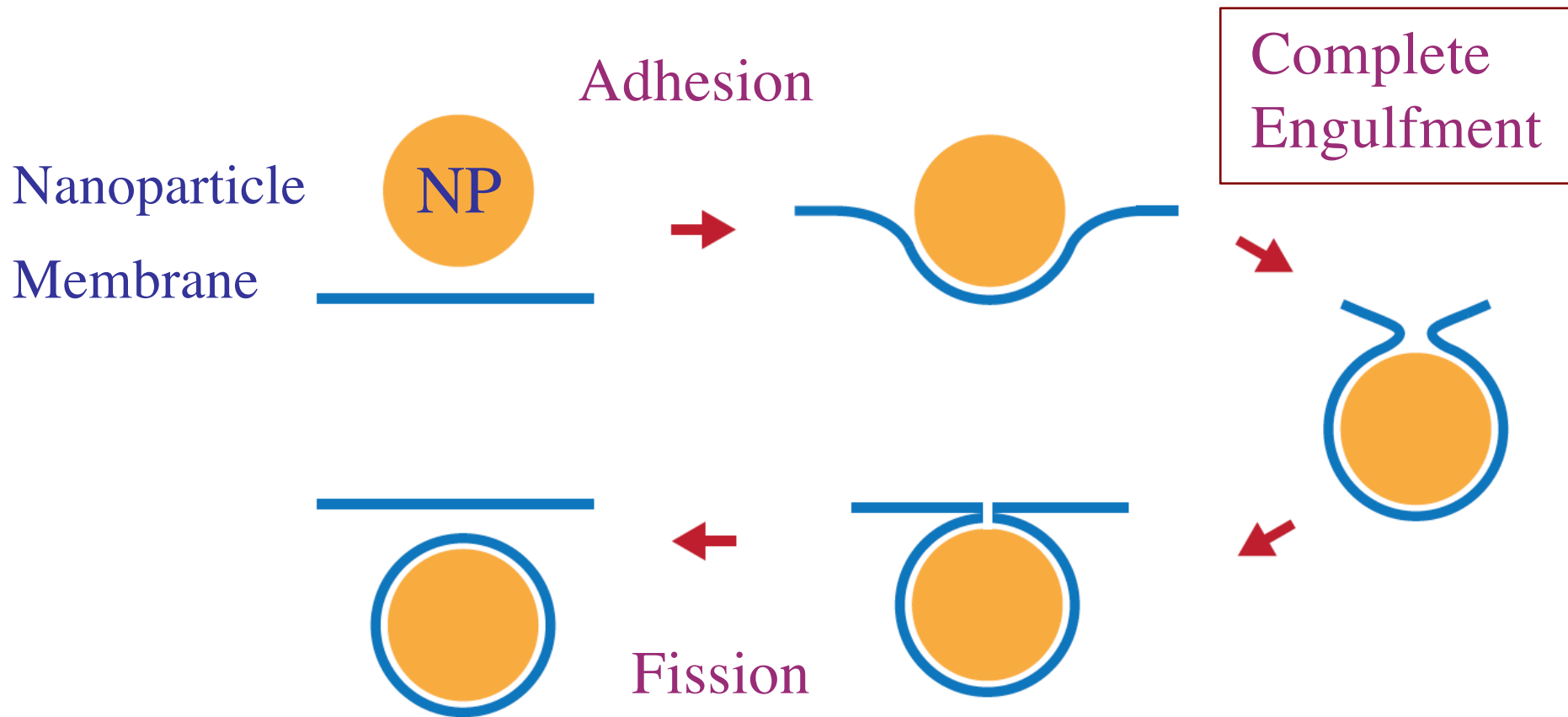


adhesion and spreading

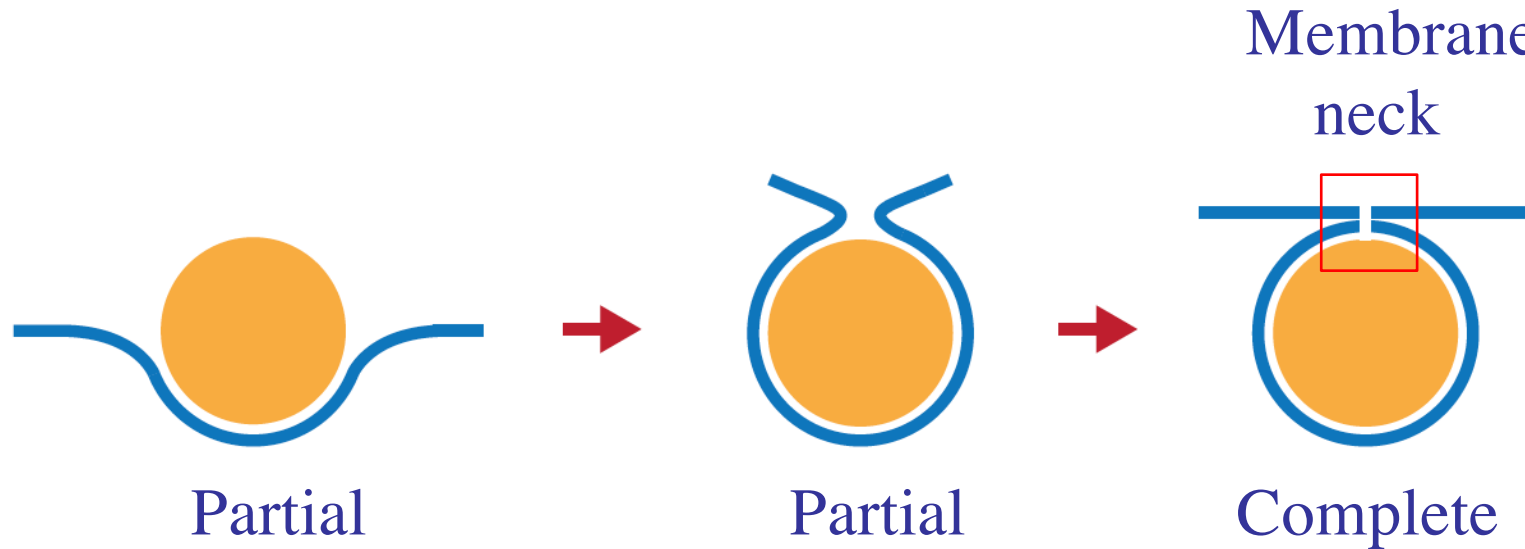


- Large contact curvature M_{co} for small R_W or large $|W|$

Endocytosis: Complete Engulfment



Engulfment: Basic Aspects



- After onset of adhesion, membrane spreads over NP
- Membrane may engulf NP only partially or completely
- Complete engulfment involves closed membrane neck
- Necessary condition for complete engulfment:
Closed membrane neck must be stable

Neck Stability: Local Criterion

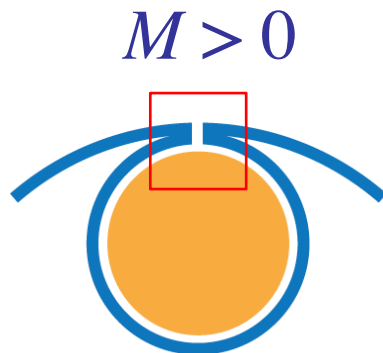
Agudo-Canalejo and RL, *ACS Nano + Nano Letters* (2015)

- Closed membrane neck is stable if membrane curvature

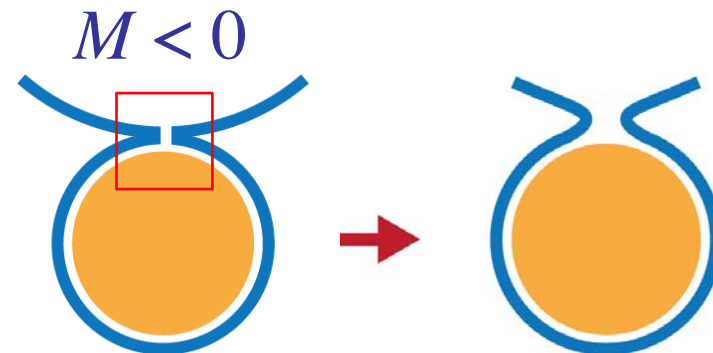
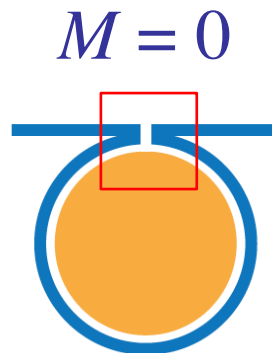
$$M \geq 2m + 1/R_{pa} - 1/R_W =: M_{ne}$$

2nd threshold
value for M

- Example: $M_{ne} = 2m + 1/R_{pa} - 1/R_W = 0$

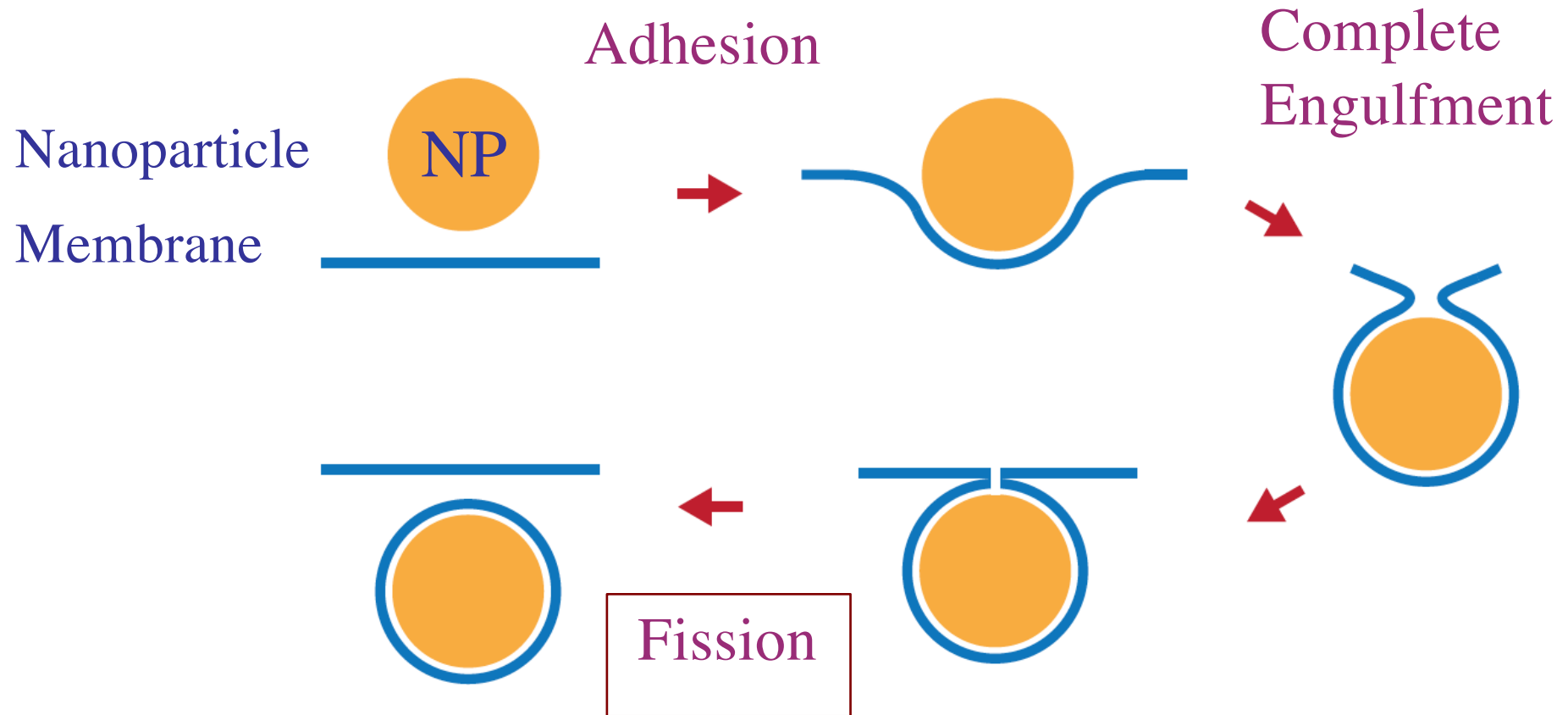


closed neck is stable



closed neck is unstable
and opens up

Endocytosis: Fission



Effective Constriction Force

- Closed neck stability with force f :

$$M + M_{co} - 2m + f (4\pi\kappa)^{-1} \geq 0$$

- Effective constriction force

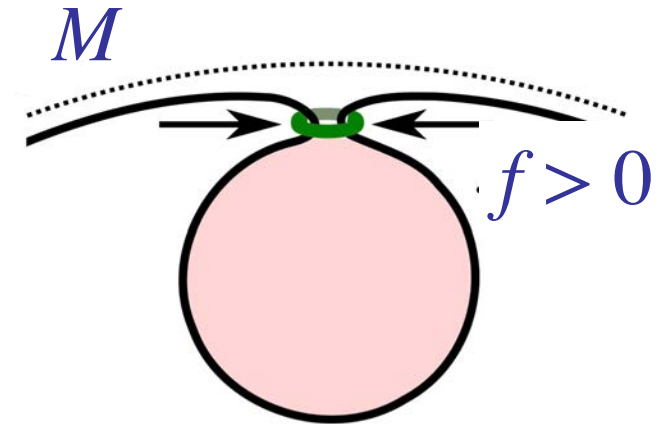
$$f_{\text{eff}} = f + 4\pi\kappa (M + M_{co} - 2m) \geq 0$$

- Engulfment force $f_{\text{eng}} = 4\pi\kappa (M + M_{co} - 2m)$

- Example: $\kappa = 4 \times 10^{-19}$ J, $M = M_{co} = 0$, $m = -1/(100 \text{ nm})$

$$\Rightarrow \text{Engulfment force } f_{\text{eng}} = 100 \text{ pN}$$

- Sufficient to create two hydrophobic bilayer edges



Receptor-Mediated Endocytosis

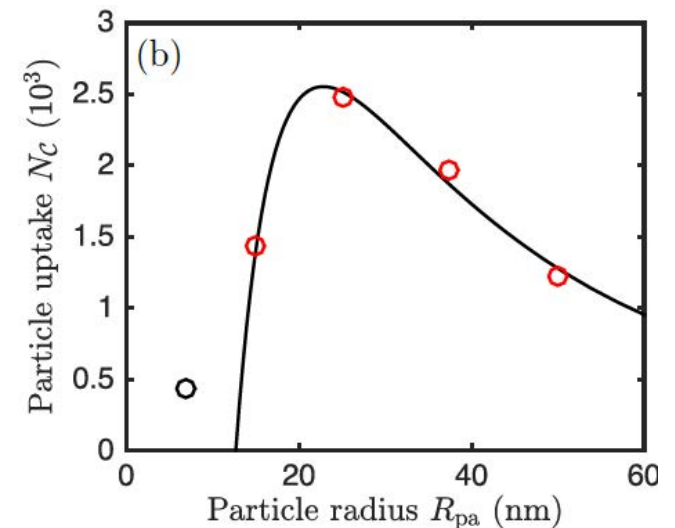
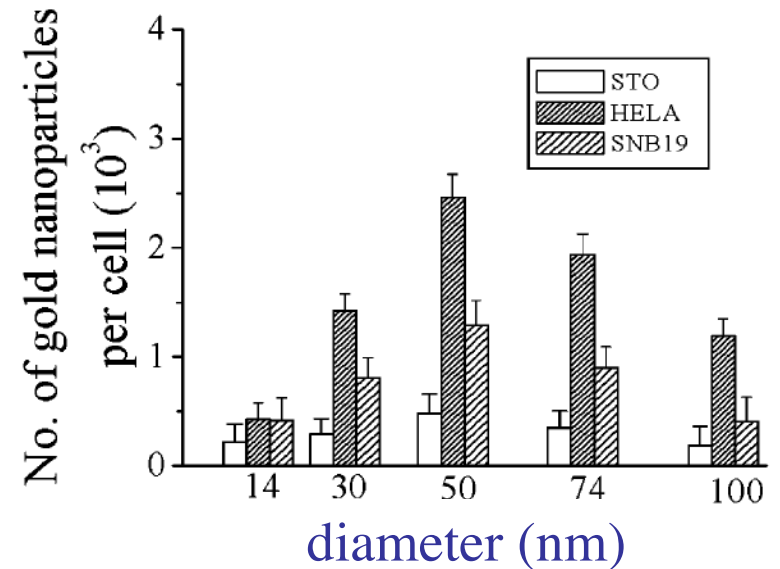
Chithrani et al, *Nano Letters* (2007)


- Uptake of gold nanoparticles by cells
- Particles bind to transferrin receptors
- Assembly of clathrin-coated vesicles

Non-monotonic size-dependence !

- Cell membrane with two types of segments, bound and unbound
- Bound segment contains protein coat with spont curv $m_{bo} = -1/(40 \text{ nm})$
- Good agreement with exp data:

Agudo-Canalejo, RL: *ACS Nano* (2015)



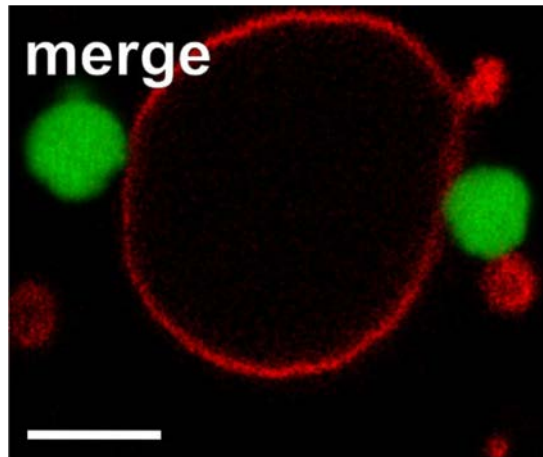
- Vesicles and Membranes
- Membrane tension and curvature elasticity
- Spontaneous curvature
- Morphological complexity of vesicles
- Outlook on related topics:
 - Bilayer membranes and leaflet tensions
 - Endocytosis of nanoparticles by membranes
 -  Wetting and engulfment of droplets

...

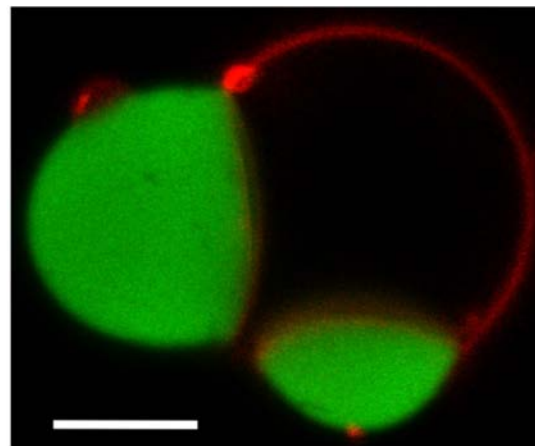
Biomolecular Condensates

Brangwynne ... Hyman, *Science* (2009)

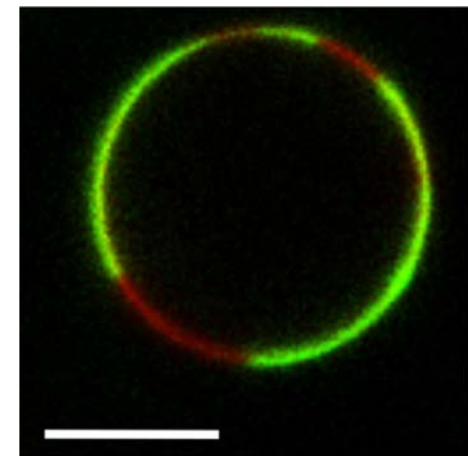
- Biomolecular condensates that behave like liquid droplets
- Enriched in intrinsically disordered proteins (IDPs)
- Example for IDP: RNA-binding protein FUS
- Interaction of FUS-droplets with GUVs, two subsequent wetting transitions:



dewetting for
high salt



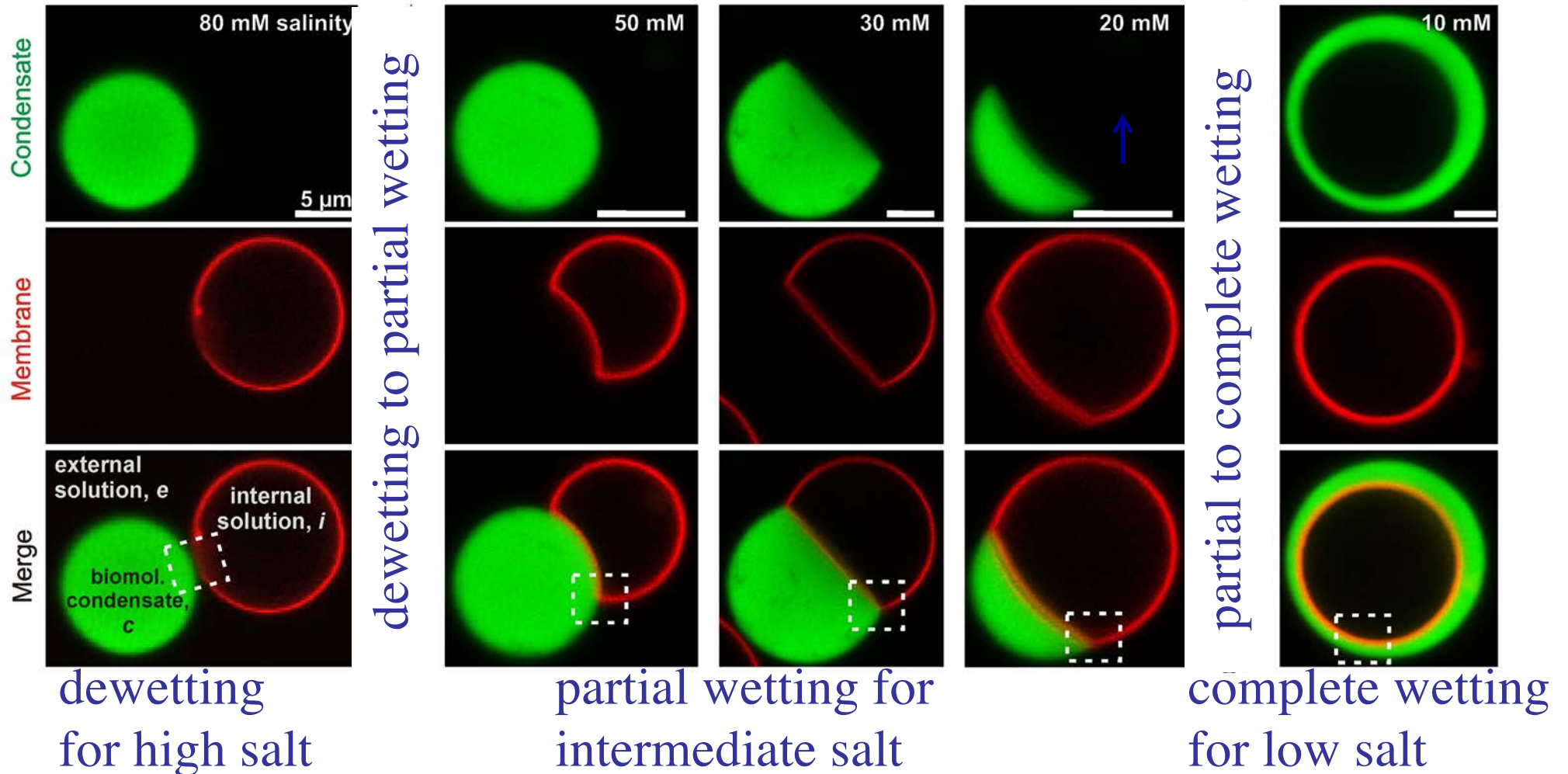
partial wetting for
intermediate salt



complete wetting
for low salt

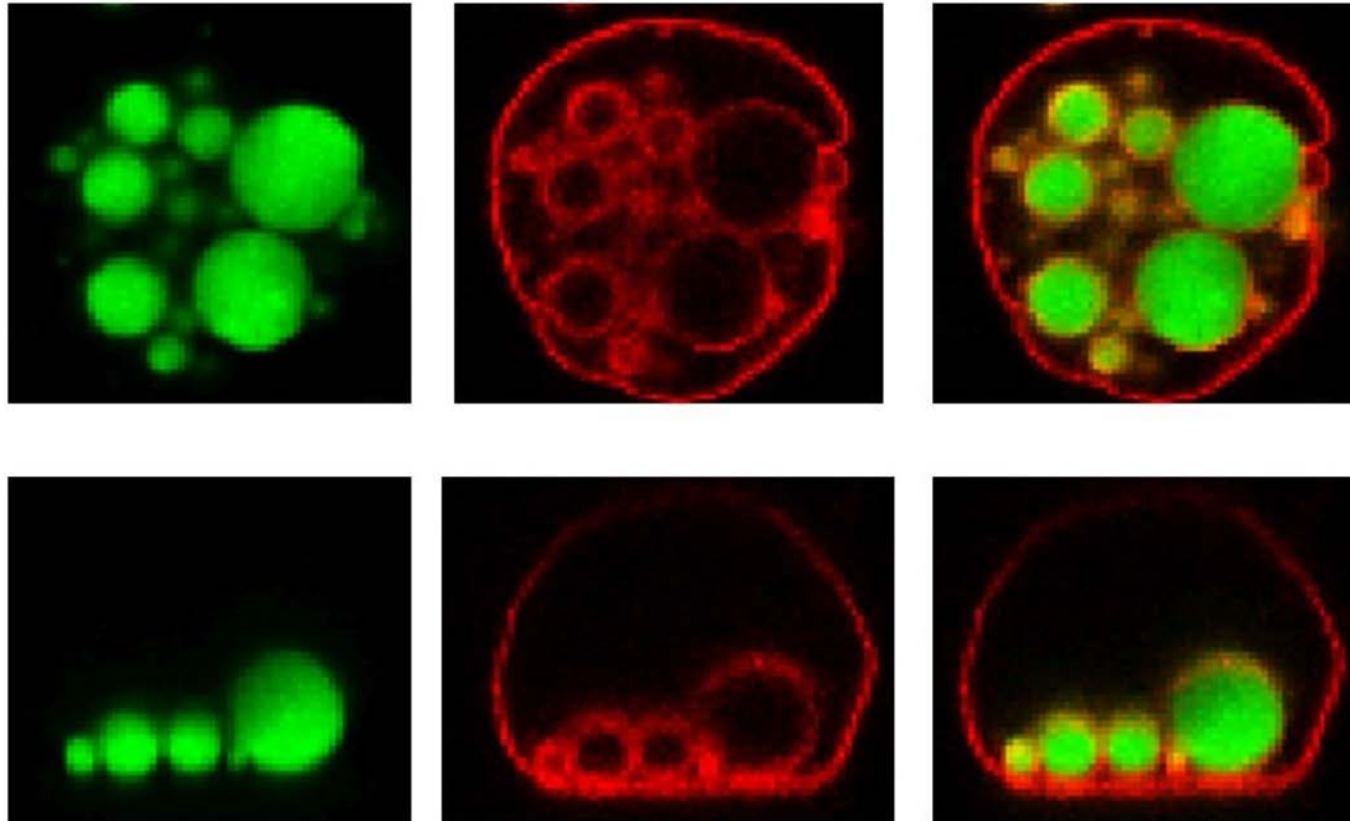
Two Wetting Transitions

- **GUV + FUS-rich organelle + salt**



Engulfment of Condensates

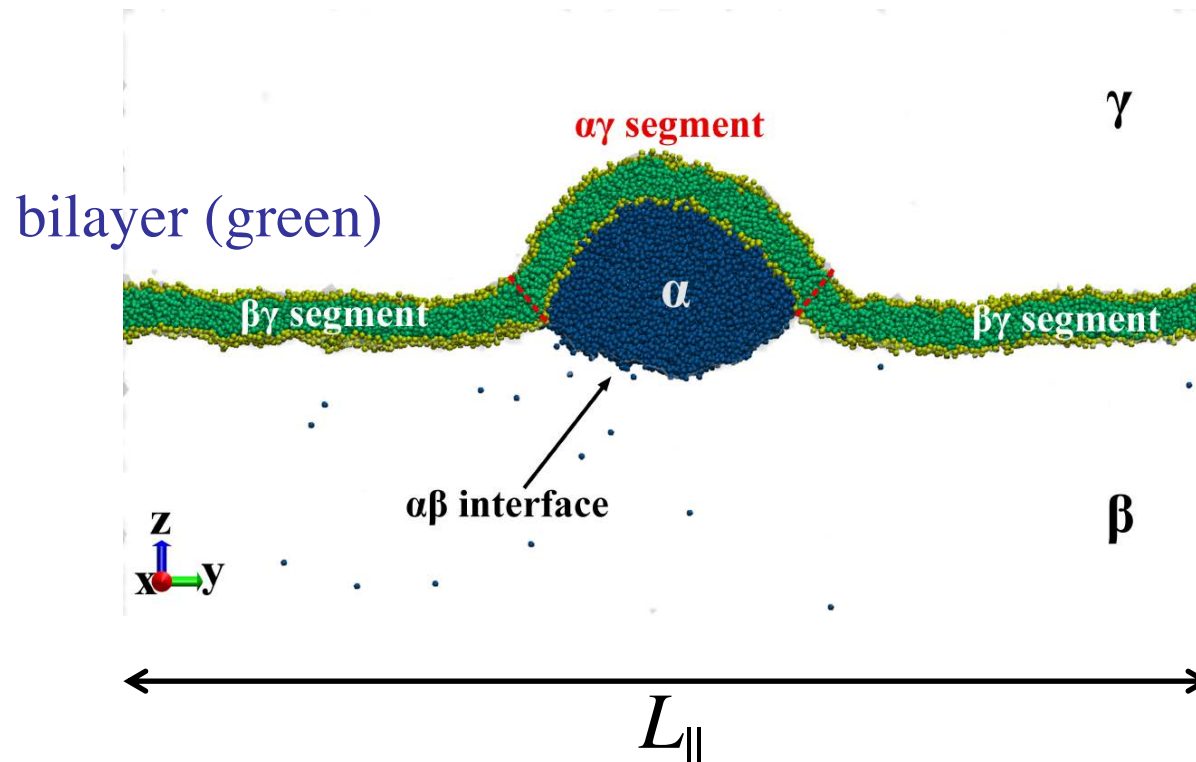
- Green FUS-rich condensates engulfed by red GUV:



Lipid Bilayer + Nanodroplet

Satarifard, Grafmüller, RL: *ACS Nano* (2018)

- Molecular simulations of lipid bilayer + nanodroplet
- Lateral box size $L_{||}$ determines mechanical tension
- Mechanical tension \sim size $L_{||}$ as control parameter



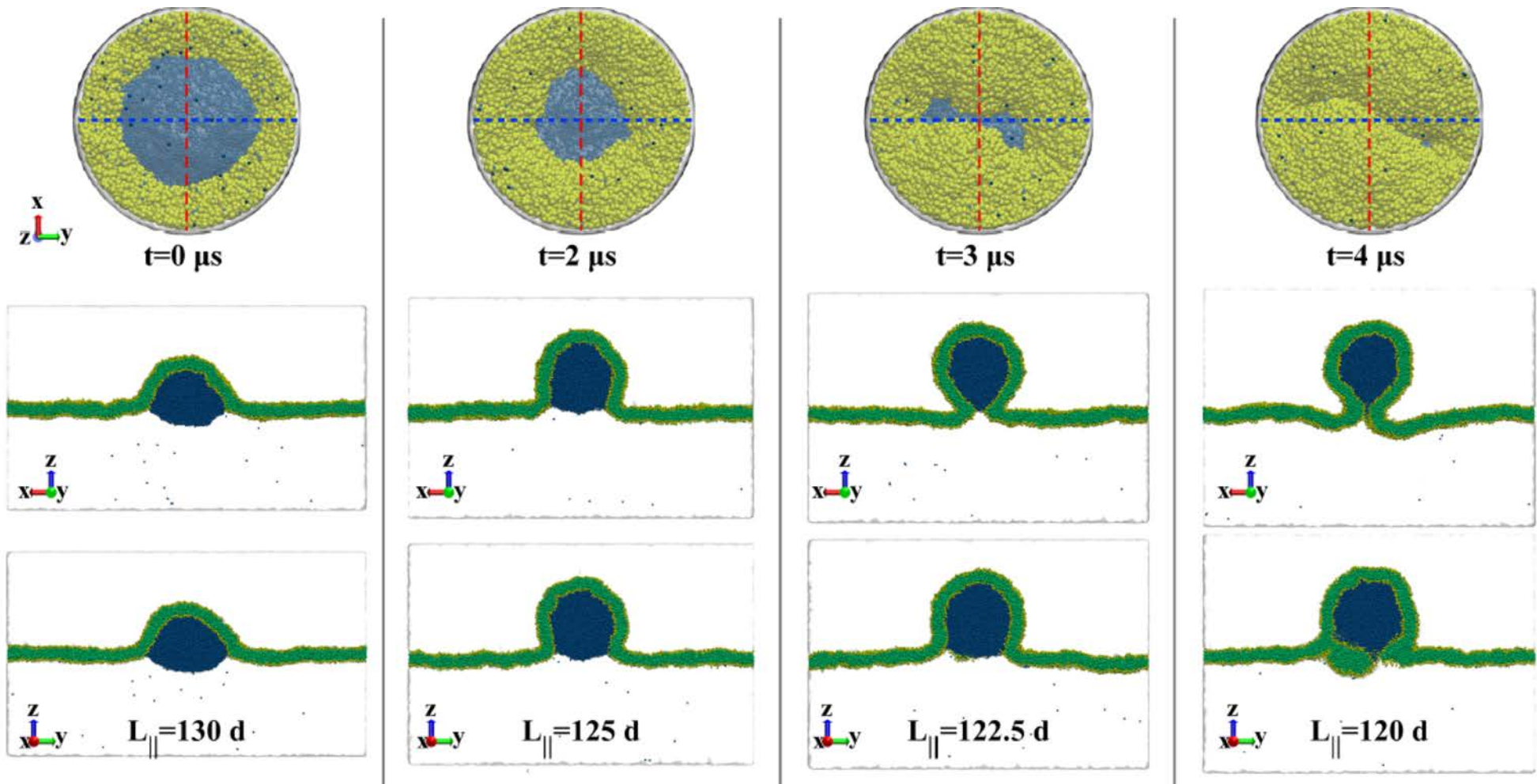
three aqueous phases α , β , γ

α droplet (blue) coexists with β phase (white)

three surface segments

$\alpha\beta$, $\alpha\gamma$, $\beta\gamma$

Engulfment of Nanodroplets



- Tight-lipped membrane neck from **negative** line tension

Summary and Outlook

- Importance of spontaneous curvature
- Morphological complexity of GUVs
 - uniform membranes \Rightarrow multi-domain membranes
- Controlled Division of GUVs
 - binding of GFP \Rightarrow photoresponsive proteins
- GUVs a la carte:
 - dsGUVs with controlled spont curvature
- Controlling morphological complexity
- Smart storage and delivery systems



- Membranes, Theo

Jaime Agudo
Andrea Grafmüller
Markus Miettinen
Aparna Sreekumari
Rikhia Ghosh
Vahid Satarifard
Simon Christ

- Membranes, Exp

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Solveig Bartelt