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Long-Range Correlations at Depinning Transitions

II. Long-Range Surface Fields

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Semi-infinite systems are considered with long-range surface fields $\propto B z^{-(1+r)}$ for large distances z from the surface. The influence of such fields on the global phase diagram and on the critical singularities of depinning transitions is studied within Landau theory. For $|B| \to 0$, the correlation length $\xi_\parallel$ diverges as $\xi_\parallel \propto b^{-1/2}$ with $b = |B| |\ln |B||^{-(1+r)}$. For finite B, $\xi_\parallel \propto |t|^{-\nu_\parallel}$ with $\nu_\parallel = (2+r)/(2+2r)$ where t measures the distance from bulk coexistence. In the latter case, a Ginzburg criterion leads to the upper critical dimension $d^* = (2+3r)/(2+r)$.

1. Introduction

In the preceding paper [1], the behaviour of the correlation length has been discussed for systems with short-range surface fields. In the work described below, long-range fields are studied which decay as $z^{-(1+r)}$ for large distances z from the surface. It is shown that these fields are a relevant perturbation at all transitions studied in [1]. In addition, the critical behaviour in the presence of long-range fields is found to depend on the "decay-exponent" r in a continuous fashion. Although this result is obtained within Landau theory, it should be valid for space dimension $d=3$ since the upper critical dimension is shown to be smaller than 3 for $r < \infty$.

The paper is organized as follows. In Sect. 2, a simplified Landau-Ginzburg model is motivated and defined. In this simplified model, the behaviour of the soft mode can be easily studied (Sect. 3). In Sect. 4 and Sect. 5, the global phase diagram and the critical behaviour along the various depinning transitions is discussed. The surface free energy is shown to have the form of an effective potential for the interface coordinate (Sect. 6). The Ginzburg criterion used in [2] for short-range forces is extended to the present case (Sect. 7). Finally, the results of Landau theory are summarized, and the changes of the phase diagram in $d=2$ are briefly discussed (Sect. 8).

2. A Simplified Landau-Ginzburg Model

In the presence of a long-range surface field $u(z)$, the Landau-Ginzburg (LG) model has the form

$$F\{\phi\} = \int d^{d-1}\rho \int_0^\infty dz \left[\tfrac{1}{2}(\nabla\phi)^2 + f(\phi) - u(z)\,\phi + \delta(z) f_1(\phi)\right]. \tag{1}$$

The surface field $u(z)$ should not affect bulk quantities which implies that $u(z) \to 0$ for $z \to \infty$. In order to obtain the usual expansion of the total free energy in terms of a bulk and a surface part, $u(z)$ must decay as (see Sect. 6)

$$u(z) \underset{z\to\infty}{\propto} z^{-(1+r)}, \qquad r > 0. \tag{2}$$

The surface term $f_1(\phi)$ is taken to be

$$f_1(\phi) = -h_1\phi + \tfrac{1}{2} a_1 \phi^2 \tag{3}$$

as in [1]. Thus, a short-range surface field h_1 is included in addition to the long-range field $u(z)$.

If the bulk term $f(\phi)$ is taken to be of the same form as in [1], the mean-field (MF) equation for the order parameter profile $M(z)$ is difficult to handle for $u(z) \neq 0$. The solution of this equation is greatly simplified, however, if one approximates the smooth double well of $f(\phi)$ (see Eq. (2) of [1]) by two parabolic pieces. The parabolic approximation for $f(\phi)$ which is investigated below has the form

$$f(\phi)=\tfrac{1}{2}(\xi^*)^{-2}\{\phi^2\,\Theta(\hat{M}-\phi) + [(1-\phi)^2+t]\,\Theta(\phi-\hat{M})\} \tag{4}$$

with

$$\hat{M}:=(1+t)/2 \tag{4.a}$$

ξ^* is the correlation length of the coexisting phases, and t measures the distance from coexistence. Parabolic approximations similar to (4) have been used before in a different context [3, 4]. They have also been studied for systems with long-range interactions by D.M. Kroll [5] and by V. Privman [6] independently from the work described here.

The MF equation follows as usually from $\delta F/\delta\phi|_{M(z)}=0$. It is convenient to introduce the dimensionless coordinate z/ξ^* which I denote by z again. This implies

$$-\ddot{M}+M=\begin{cases}(\xi^*)^2\,u(\xi^* z) & (5.a)\\ 1+(\xi^*)^2\,u(\xi^* z) & (5.b)\end{cases}$$

which has to be solved with the boundary conditions

$$\dot{M}|_{z=0}=-\xi^* h_1+\xi^* a_1 M_1 \tag{6.a}$$

$$M(z=\infty)=M_b \tag{6.b}$$

$M_1:=M(z=0)$ and M_b are the surface and bulk order parameter, respectively. A dot indicates a derivative with respect to z. In order to simplify the formulas, I will put $\xi^*\equiv 1$.

3. Soft Mode of Gaussian Fluctuations

The potential $Q(z)$ for the Gaussian fluctuations (see Eq. (9) and Eq. (11) of [1]) is particularly simple for the LG-potential (4) since

$$f''(\phi)=1-\delta(\phi-\hat{M})$$

in this case. This implies

$$Q(z)=f''(M(z))=1-\delta(M(z)-\hat{M}) = 1-|\dot{M}(\hat{\ell})|^{-1}\,\delta(z-\hat{\ell}) \tag{7}$$

where $\hat{M}=M(z=\hat{\ell})$ has been used. $\hat{\ell}$ is the MF position of the interface between the two (almost) coexisting phases (compare [1]). Thus, apart from a constant, $Q(z)$ is just a δ-function well. As a consequence, the soft mode $g_0(z)$ satisfies the Schrödinger-type equation

$$\left[-\frac{d^2}{dz^2}+1-|\dot{M}(\hat{\ell})|^{-1}\,\delta(z-\hat{\ell})\right] g_0(z)=E_0\, g_0(z) \tag{8.a}$$

with the boundary condition

$$\left.\frac{d}{dz}g_0(z)\right|_0=a_1\, g_0(0) \tag{8.b}$$

(compare Eq. (10) of [1]). An elementary matching procedure at $z=\hat{\ell}$ yields

$$g_0(z)=\frac{1}{N}\begin{cases}[2W|\dot{M}(\hat{\ell})|-1]\,e^{W(\hat{\ell}-z)}+e^{W(z-\hat{\ell})}, & z<\hat{\ell}\\ 2W|\dot{M}(\hat{\ell})|\,e^{W(\hat{\ell}-z)}, & z>\hat{\ell}\end{cases} \tag{9}$$

with a normalization constant N and

$$W:=(1-E_0)^{1/2}. \tag{10}$$

If the expression (9) is inserted into the boundary condition (8.b), one obtains the following implicit equation for E_0:

$$2W|\dot{M}(\hat{\ell})|-1=\frac{W-a_1}{W+a_1}\,e^{-2W\hat{\ell}}. \tag{11}$$

Thus, (11) determines the divergence of the correlation length $\xi_\parallel=E_0^{-1/2}$ from the asymptotic behaviour of $\hat{\ell}$ and $\dot{M}(\hat{\ell})$. As soon as the limiting behaviour of $\xi_\parallel$ has been found in this way, one may use all formulas for the correlation function $C(\rho, zz')$ which have been derived in Sect. 5 of [1]. This implies that the interface is rough in $d\leqq 3$ for any r.

4. Extended Phase Diagram

So far, the functional form of $u(z)$ has not been specified apart from its asymptotic behaviour (2). In order to simply the determination of $\hat{\ell}$, I will choose

$$u(z)=B(z+\Delta)^{-(1+r)}[1-(1+r)(2+r)(z+\Delta)^{-2}] \tag{12}$$

with $r>0$ and

$$\Delta>[(1+r)(2+r)]^{1/2}. \tag{12.a}$$

The inequality (12.a) ensures that $u(z)$ does not change sign for $0<z<\infty$. For large z,

$$u(z)\to B z^{-(1+r)}, \quad r>0. \tag{12.b}$$

For the above choice of $u(z)$, a special solution of the inhomogeneous differential equation (5) can be obtained without the use of Green's function techniques. As a consequence, various MF quantities can be calculated analytically. As shown below, their critical behaviour is characterized by surface exponents which depend on r but not on Δ. This is consistent with the expectation that the detailed form of $u(z)$ should be unimportant for the leading terms of the critical singularities.

The model (1)-(4) with $u(z)$ given by (12) depends on the bulk variable t and the surface variables h_1, a_1 and B. The phase boundaries in the $(t; h_1, a_1, B)$-space are most easily envisaged if one starts with the $(t=0;\ h_1,\ a_1,\ B=0)$-subspace. In this case, the qualitative features of the phase diagram displayed in Fig. 2 of [1] are recovered from model (1)-(4). In fact, the coordinates of the depinning transitions $(C^\pm)$ and $(T^\pm)$ which follow from the parabolic approximation (4) for $f(\phi)$ are the same as those for the smooth double well potential discussed in [1] with $p=1$.

For $B \neq 0$ and $t=0$, two surfaces of discontinuous transitions $(LD^\pm)$ are found (the letter L means "long-range", the letter D "discontinuous"). The surface (LD^-) is attached to the (h_1, a_1)-plane along the phase boundaries (D^-) and (C^-) (see Fig. 2 of [1]), and extends into the region with $B<0$. Similarly, the surface (LD^+) is attached to the (h_1, a_1)-plane along (D^+) and (C^+), and extends into the region with $B>0$.

In Fig. 1, a 2-dim. intersection of the $(t=0;\ h_1,\ a_1,\ B)$-space is shown for $a_1 = \text{const} > 1$. Depinning transitions occur in the shaded region of Fig. 1 denoted by $(LP^\pm)$, and along their boundaries $(P^\pm)$ and $(C^\pm)$. For $B \to 0^-$, the phase boundary (LD^-) (see Fig. 1) is given by

$$h_1 = \tilde{h}_1 = c_0 |B|^{1/2} |\ln|B||^{-r/2} \tag{13}$$

with

$$c_0 = 2^{r/2} r^{-1/2} (a_1^2 - 1)^{1/2}. \tag{13.a}$$

Similarly, the phase boundary (LD^+) is found to be

$$h_1 = \tilde{\tilde{h}}_1 = a_1 - c_0 B^{1/2} |\ln(B)|^{-r/2} \tag{14}$$

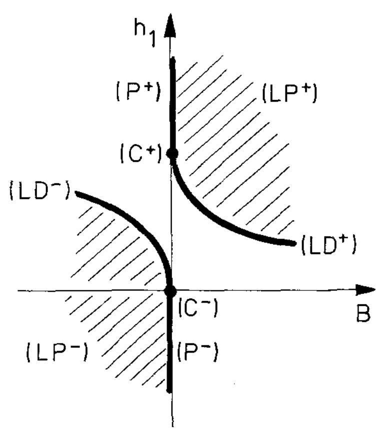

Fig. 1. Phase boundaries in the (h_1, B)-subspace for $t=0$ and $a_1>1$. (LP), (P), and (C) are three different types of depinning transitions. $A(-)$ and $a(+)$ indicate that depinning occurs for $t \to 0^-$ and $t \to 0^+$, respectively

for $B \to 0^+$. The derivation of these formulas is explained in Sect. 6.

Along $(LD^\pm)$, the interface position $\hat{\ell}$ jumps from a finite value to infinity. In addition, the hierarchy of continuous transitions discussed in [1] is extended by the additional transitions $(LP^\pm)$. Thus, in the phase diagram of Fig. 1 which applies for $a_1>1$, three types of continuous transitions are distinguished by their number of scaling fields: one, two and three such fields are present at (LP), (P), and (C), respectively. Note that the type of surface criticality at (P) and (C) has been changed by the inclusion of long-range surface fields since B is an additional relevant perturbation at these transitions.

For $a_1=1$, the depinning transitions $(C^\pm)$ are replaced by the transitions $(T^\pm)$ (see Fig. 2 of [1]). In the enlarged phase diagram, $(T^\pm)$ has four relevant scaling fields. However, due to the parabolic approximation of $f(\phi)$, the critical singularities at $(T^\pm)$ are not correctly described by model (1)-(4). Similarly, one can not find higher-order multicritical transitions such as (Q^-) (see [1]) from this model.

In the context of wetting, (LD^+) separates the region of complete wetting $(=(LP^+))$ from the region of incomplete wetting in the presence of long-range substrate fields. The fact that the transition along this phase boundary is discontinuous has also been found for an effective interface model from a real-space renormalization group method [7].

So far, the discussion of the phase diagram has been restricted to bulk coexistence i.e. $t=0$. For $t \neq 0$, there are two wings of first-order transitions which are attached to the (h_1, B)-plane of Fig. 1 along $(LD^\pm)$. The wing at (LD^-) and (LD^+) extend into the region with $t<0$ and $t>0$, respectively. As $|B| \to 0$, the edges of these wings which are second-order transitions approach the (h_1, B)-plane. For instance, the edge of the (LD^-) wing has coordinates $\bar{t}(B)$ and $\bar{h}_1(B)$ in the $(t; h_1, B)$-subspace which behave asymptotically as

$$\bar{t} \propto -|B| |\ln|B||^{-(1+r)}$$

$$\bar{h}_1 \propto |B|^{1/2} |\ln|B||^{-(1+r/2)}.$$

5. Critical Behaviour

Apart from a redefinition of the scaling fields, the critical properties of the transitions with $(+)$ and with $(-)$ (see Fig. 1) are identical. Therefore, only the SID transitions with $(-)$ will be explicitly discussed below. For SID, the MF equations (5) and (6) have to be solved with the boundary condition

$M(z=\infty)=M_0\equiv 1$. This leads to

$$M(z)=\begin{cases}\frac{1}{2}e^{z-\hat{\ell}}+[M_1-\frac{1}{2}e^{-\hat{\ell}}-B\Delta^{-(1+r)}]e^{-z}\\ \quad +B(z+\Delta)^{-(1+r)} \qquad z<\hat{\ell}\\ 1+[-\frac{1}{2}+\frac{1}{2}t-B(\hat{\ell}+\Delta)^{-(1+r)}]e^{\hat{\ell}-z}\\ \quad +B(z+\Delta)^{-(1+r)} \qquad z>\hat{\ell}.\end{cases} \tag{15}$$

In (15), $\hat{\ell}$ and M_1 enter as parameters which have to be determined from the boundary condition $\dot{M}(0)=-h_1+a_1 M_1$ and from the matching condition $M(\hat{\ell})=\hat{M}=(1+t)/2$. As a result, one obtains

$$\tfrac{1}{2}|t|+BX_B(\hat{\ell})+(a_1+1)^{-1}h_1 e^{-\hat{\ell}} -\tfrac{1}{2}(a_1+1)^{-1}(a_1-1)e^{-2\hat{\ell}}=0 \tag{16}$$

with

$$X_B(\ell)=(\ell+\Delta)^{-(1+r)}-c_1 e^{-\ell} \tag{17}$$

$$c_1:=\Delta^{-(1+r)}[a_1+(1+r)/\Delta]/[a_1+1]>0 \tag{17.a}$$

as an implicit equation for $\hat{\ell}$. Due to the general inequality $e^x>(1+x/y)^y$ [8], $X_B(\ell)>0$ for $0<\ell<\infty$ as long as r and Δ satisfy (12.a). In terms of $\hat{\ell}$, the surface order parameter M_1 is given by

$$M_1=\left[h_1+\Delta^{-(1+r)}\left(1-\frac{1+r}{\Delta}\right)B\right]\Big/[a_1+1]+\delta M_1 \tag{18.a}$$

$$\delta M_1:=e^{-\hat{\ell}}/(a_1+1). \tag{18.b}$$

It is useful to consider δM_1 as the singular part of M_1. In order to determine the correlation length $\xi_\parallel$ via (11), one needs $\dot{M}(\hat{\ell})$. From (15),

$$\dot{M}(\hat{\ell})=\tfrac{1}{2}+\delta\dot{M}(\hat{\ell}) \tag{19.a}$$

$$\delta\dot{M}(\hat{\ell}):=-t/2+B(\hat{\ell}+\Delta)^{-(1+r)}-B(1+r)(\hat{\ell}+\Delta)^{-(2+r)}. \tag{19.b}$$

If (19) is inserted into (11) and the resulting expression is expanded in powers of E_0, the leading order terms lead to

$$\xi_\parallel^{-2}=E_0=4[\delta\dot{M}(\hat{\ell})+\tfrac{1}{2}(a_1+1)^{-1}(a_1-1)e^{-2\hat{\ell}}] \tag{20}$$

(16), (18), and (20) determine the critical behaviour of $\hat{\ell}$, M_1 and $\xi_\parallel$.

First, consider the transition (LP^-) which occurs for $t=0^-$, $B<0$, and $h_1<\hat{h}_1$ (see Fig. 1 and (13)). From (16), one finds the power law divergence

$$\hat{\ell}=A_1|t|^{\beta_s} \tag{21}$$

for the interface position with

$$\beta_s=-1/(1+r) \tag{21.a}$$

$$A_1=(2|B|)^{-\beta_s}. \tag{21.b}$$

The surface exponent (21.a) has also been found in the context of lattice gas models [9]. If (21) is inserted into (18), one sees immediately that δM_1 has an essential singularity:

$$\delta M_1\propto\exp(-A_1|t|^{\beta_s}). \tag{22}$$

Finally, (20), (16), and (21) imply a power law divergence for the correlation length:

$$\xi_\parallel=A_2|t|^{-\nu_\parallel} \tag{23}$$

with

$$\nu_\parallel=(2+r)/(2+2r) \tag{23.a}$$

$$A_2=\tfrac{1}{2}[|B|(1+r)]^{-1/2}[2|B|]^{\nu_\parallel}. \tag{23.b}$$

The surface exponent (23.a) has been obtained previously in the context of effective interface models [10]. Note that the surface exponents β_s and $\nu_\parallel$ depend on r in a continuous way. In contrast, both the exponents and the amplitudes in (21)-(23) which are the leading terms do not depend on the parameter Δ (see (12)). If next-to-leading terms are considered, one finds that Δ enters their amplitudes.

The SID transition (P^-) occurs at $t=B=0$ (see Fig. 1, and Fig. 2 of [1]). The leading terms of $\hat{\ell}$, δM_1, and $\xi_\parallel$ found from (16)-(20) in this case may be written in a scaling form. There are two scaling fields, namely $|t|$ and

$$b:=|B|\,|\ln|B||^{-(1+r)} \tag{24}$$

rather than B itself. In terms of t and b, the asymptotic behaviour of the surface order parameter is

$$\delta M_1=|t|\,\Omega_1(|t|^{-\Delta_b}b) \tag{25}$$

with

$$\Omega_1(x)\propto x, \qquad x\to\infty. \tag{25.a}$$

At (P), the new surface exponent Δ_b has the MF value

$$\Delta_b=1. \tag{26}$$

From (18),

$$\hat{\ell}=-\ln[|t|\,\Omega_1(|t|^{-\Delta_b}b)]. \tag{27}$$

The asymptotic behaviour of the correlation length is

$$\xi_\parallel=|t|^{-1/2}\,\Omega_2(|t|^{-\Delta_b}b) \tag{28}$$

with

$$\Omega_2(x)\propto x^{-1/2}, \qquad x\to\infty. \tag{28.a}$$

Thus, for $t=0$ and $B \to 0^+$, the correlation length diverges as $\xi_\parallel \propto b^{-1/2} = B^{-1/2} |\ln(B)|^{(1+r)/2}$.

Finally, consider the continuous transition (C^-) which occurs for $B=t=h_1=0$ and $a_1>1$ (see Fig. 1, and Fig. 2 of [1]). At (C^-) one has three scaling fields, namely $|t|$, b as given by (24), and h_1. In this case, the surface order parameter is found to have the scaling form

$$\delta M_1 = |t|^{1/2} \psi_1(|t|^{-\Delta_b} b, |t|^{-\Delta_1} h_1) \tag{29}$$

with

$$\psi_1(x,0) \propto x^{1/2}, \qquad x \to \infty \tag{29.a}$$

$$\psi_1(0,y) \propto y, \qquad y \to \infty. \tag{29.b}$$

The surface exponents Δ_b and Δ_1 have the MF values

$$\Delta_b = 1 \tag{30}$$

$$\Delta_1 = 1/2. \tag{31}$$

Thus, $\Delta_b=1$ both at (P) and at (C). Δ_1 has been determined before [2, 1]. The divergence of the interface position $\hat{\ell}$ follows from (18.b) and (29), and the critical behaviour of the correlation length is

$$\xi_\parallel = |t|^{-1/2} \psi_2(|t|^{-\Delta_b} b, |t|^{-\Delta_1} h_1) \tag{32}$$

with

$$\psi_2(x,0) \to x^{-1/2}, \qquad x \to \infty \tag{32.a}$$

$$\psi_2(0,y) \to 1/y, \qquad y \to \infty \tag{32.b}$$

at (C^-).

6. Surface Free Energy

In the context of Landau theory, the surface free energy f_s for the model (1) reads

$$f_s = \int_0^\infty dz \left[\frac{1}{2}\left(\frac{dM}{dz}\right)^2 + f(M) - f(M_b) - uM\right] + f_1(M_1). \tag{33}$$

In (33), the term $u(z)M(z)$ has the large z-behaviour $Bz^{-(1+r)} M_b$. Therefore, its contribution to f_s is finite only for $r>0$. For $-1<r\leqq 0$, the total free energy contains a term which is intermediate between the bulk and the surface free energy.

If the MF profile (15) is inserted into (33), one may treat $\hat{\ell}$ as an independent variable. As a result, one obtains after some tedions but straightforward calculations

$$f_s = V(\hat{\ell}) + \tfrac{1}{4} + O(B^2, Bh_1, h_1^2) \tag{34}$$

with

$$V(\ell) = \tfrac{1}{2}|t|\ell + BY_B(\ell) - (a_1+1)^{-1} h_1 e^{-\ell} + \tfrac{1}{4}(a_1+1)^{-1}(a_1-1)e^{-2\ell} \tag{35}$$

$$Y_B(\ell) = -\frac{1}{r}(\ell+\Delta)^{-r} + c_1 e^{-\ell}, \tag{35.a}$$

and $dY_B(\ell)/d\ell = X_B(\ell)$ (cf. (17)). Since $-Y_B(\ell) > X_B(\ell) > 0$ due to (12.a), it follows that $Y_B(\ell)<0$ for $0<\ell<\infty$.

The "effective potential" $V(\ell)$ has two nice properties:

1) the implicit equation (16) for the interface position $\hat{\ell}$ is identical with

$$\partial V(\ell)/\partial \ell|_{\hat{\ell}} = 0 \tag{36}$$

2) the leading term (20) for the correlation length $\xi_\parallel$ is recovered from

$$\xi_\parallel^{-2} = 4 \left.\frac{\partial^2 V(\ell)}{\partial \ell^2}\right|_{\hat{\ell}} = 4[\delta \dot{M}(\hat{\ell}) + \tfrac{1}{2}(a_1+1)^{-1}(a_1-1)e^{-2\hat{\ell}}]. \tag{37}$$

The implicit equation (36) (or (16)) may have several solutions. In particular, at the discontinuous transition (LD^-), the solution $\hat{\ell}=\infty$ coexists with the solution $\hat{\ell}=\tilde{\ell}$ since $V(\infty)=V(\tilde{\ell})$ which implies $f_s(\hat{\ell}=\infty)=f_s(\hat{\ell}=\tilde{\ell})$ due to (34). The schematic form of $V(\ell)$ at (LD^-) is shown in Fig. 2. As a consequence, (LD^-) is determined from

$$V(\tilde{\ell})=0, \qquad \partial V/\partial \ell|_{\tilde{\ell}}=0,$$

which has to be solved for $\tilde{\ell}(a_1, B)$ and $\tilde{h}_1(a_1, B)$. For $B\to 0^-$, the solution $\tilde{h}_1$ has the asymptotic behaviour given by (13).

If the asymptotic behaviour of $\hat{\ell}$ is inserted into (34), one obtains the singular part of the surface free energy f_s. At the depinning transition (LP^-), the leading term is

$$f_s = A_3 |t|^{2-\alpha_s} \tag{38}$$

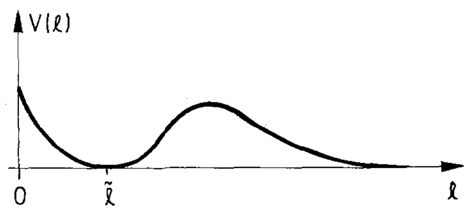

Fig. 2. The effective potential $V(\ell)$ at the transition (LD^-), cf. Eq. (35). The phases with $\hat{\ell}=\tilde{\ell}$ and $\hat{\ell}=\infty$ coexist in this case

with

$$2-\alpha_s=r/(1+r) \tag{38.a}$$

$$A_3=\tfrac{1}{2}(2|B|)^{-\beta_s}-\frac{1}{r}B(2|B|)^{r\beta_s}. \tag{38.b}$$

Note that α_s and A_3 are again independent of the parameter Δ. There are two types of next-to-leading terms for f_s: *1)* terms $\propto\hat{\ell}^{-(n+r)}\propto|t|^{(n+r)/(1+r)}$ with $n=1,2,\ldots$; *2)* essential singularities due to the $e^{-\hat{\ell}}$-factors in (35).

At the depinning transitions (P^-) and (C^-), f_s depends on the scaling fields $|t|, b$, and $|t|, b, h_1$, respectively. The dependence of f_s on $|t|$ and h_1 has been discussed previously [2]. The dependence on the new scaling field b is

$$f_s \propto b\ln(b). \tag{39}$$

7. Ginzburg Criterion

A simple Ginzburg criterion can be obtained [2] if one compares the MF contribution f_s to the surface free energy with

$$\Delta f_s \propto \begin{cases} E_0\ln(1/E_0), & d=3 \\ E_0^{(d-1)/2}, & d\neq 3 \end{cases} \tag{40}$$

which is the contribution due to the soft mode. In field-theoretic terminology, f_s is a zero-loop term and Δf_s is a one-loop term [cf. 11]. This Ginzburg criterion has been used before to determine the upper critical dimension $d^*=3$ for all depinning transitions in systems with short-range forces (i.e. $B=0$) [10]. Thus, the critical behaviour at the transitions (P), (C), (T), and (Q) (see Fig. 1 and Fig. 2 of [1]) is correctly described by MF theory for $d>3$.

In the presence of long-range forces ($B\neq 0$), the system may undergo the continuous transition (LP). In this case, one has

$$f_s \propto |t|^{2-\alpha_s}, \quad 2-\alpha_s=r/(r+1) \tag{41}$$

from (38) and

$$\Delta f_s \propto |t|^{(d-1)\nu_\parallel}, \quad \nu_\parallel=(2+r)/(2+2r) \tag{42}$$

from (23) and (40). At the upper critical dimension d^*, (41) and (42) are equally important. This implies

$$d^*(r)=(2+3r)/(2+r). \tag{43}$$

Thus, $d^*<3$ for $0<r<\infty$. In particular, (43) yields

$$d^*(r=2)=2. \tag{43.a}$$

The case of short-range surface fields is recovered from (43) for $r\to\infty$ since $d^*(r\to\infty)=3$.

Note that this Ginzburg criterion is equivalent to the simple rule that d^* follows from the hyperscaling relation $2-\alpha_s=(d-1)\nu_\parallel$ when the MF values for α_s and $\nu_\parallel$ are inserted. This rule has been used in [10] in order to obtain the upper critical dimension (43) from an effective interface model. On the other hand, the Ginzburg criterion used above is a stability criterion of the usual form since one-loop terms are compared with zero-loop terms [cf. 11]. In this sense, the Ginzburg criterion provides a justification for the simple hyperscaling-rule.

8. Systematics of Surface Criticality

In the preceding paper [1], four different types of depinning transitions denoted by (P), (C), (T), and (Q) have been discussed. In the present work, long-range surface fields (with amplitude B) were shown to be a relevant perturbation at (P) and (C). Due to the parabolic approximation (4), (T) and (Q) could not be treated for $B\neq 0$ but it is quite obvious that B is also relevant in these cases. In addition, the continuous depinning transition (LP) can occur in the presence of long-range fields i.e. for $B\neq 0$ (see Fig. 1). In summary, a *hierarchy of five different types* of depinning transitions has been discussed: (LP), (P), (C), (T), and (Q) with one, two, ..., and five relevant scaling fields, respectively.

The upper critical dimension is $d^*=(2+3r)/(2+r)$ for (LP) (see (43)) and $d^*=3$ for all other transitions [2, 10]. For $d>d^*$, the results of MF or Landau theory described above should be valid.

At all transitions, the bulk variable t which measures the distance from bulk coexistence is a relevant perturbation. For $d>d^*$, the t-dependence of the critical behaviour is characterized by two independent exponents α_s and $\nu_\parallel$ which govern the singularities of the surface free energy and the correlation

Table 1. Independent surface exponents at the various depinning transitions (LP), (P), (C), $(T^\pm)$, and (Q^-) above their upper critical dimensions. The parameter r determines the behaviour of the long-range surface field (see Eq. (2)), the parameter p enters the bulk potential (see Eq. (2) of [1])

	α_s	$\nu_\parallel$	Δ_b	Δ_1	ϕ_a	ϕ_b
(LP)	$\frac{2+r}{1+r}$	$\frac{2+r}{2+2r}$				
(P)	1	1/2	1			
(C)	1	1/2	1	1/2		
(T^-)	1	1/2	1	$\frac{p+1}{p+2}$	$\frac{p}{p+2}$	
(T^+)	1	1/2	1	2/3	1/3	
(Q^-)	1	1/2	1	$\frac{2p+1}{2p+2}$	$\frac{2p}{2p+2}$	$\frac{p}{2p+2}$

length, respectively. For $d \leqq d^*$, α_s and $\nu_{\parallel}$ are related by the hyperscaling relation $2-\alpha_s=(d-1)\nu_{\parallel}$. At $(P), (C), \ldots$, the relevant scaling field (24) related to long-range surface fields can be characterized by the scaling index Δ_b. Its MF value is $\Delta_b=1$. At (C), (T), and (Q), short-range surface fields characterized by Δ_1 become important. Finally, at (T) and (Q), there are additional relevant perturbations related to local surface operators as discussed previously [12]. All independent surface exponents for $d>d^*$ are collected in Table 1.

For $d \leqq d^*$, the phase boundaries and the critical singularities are changed by fluctuations. This can be shown explicitly for $d=2$. In this case, effective interface models can be studied by transfer-matrix methods. In the presence of both long-range and short-range surface fields, the expression (35) may be used as an effective potential $V(\ell)$ for the interface coordinate ℓ. In the transfer-matrix formalisme, $V(\ell)$ becomes a "quantum-mechanical" potential in a Schrödinger-type equation. The ground-state energy of this potential yields the surface free energy, and the ground-state wave function determines all moments of the interface coordinate ℓ [13].

For $d=2$, special cases of the effective potential (35) have been investigated in [13] and [10]. The results obtained in these references may be extended to yield the phase diagram in the (h_1, B)-subspace. For $r>2$, one concludes that (P), (C), and (LD) merge into one phase boundary which may be denoted by (PC). As a consequence, there are only two types of depinning transitions, namely (LP) and (PC) (cf. Fig. 1). The transition line (PC^-), for instance, intersects the h_1-axis at $h_1>0$, and the B-axis at $B>0$ [13]. Thus, the region of the (h_1, B)-plane where the depinning transition (LP^-) occurs is increased by fluctuations. The surface exponents at (LP) and (PC) with $r>2$ are given in Table 2a.

Table 2. Surface exponents at depinning transitions in $d=2$ in the presence of long-range surface fields with a) $r>2$, and b) $0<r<2$ [13, 10]. Note that hyperscaling holds at all transitions except for (LP) with $0<r<2$

	α_s	$\nu_{\parallel}$	$\Delta_b=\Delta_1$
(LP)	4/3	2/3	
(PC)	4/3	2/3	1/3

a) $r>2$

	α_s	$\nu_{\parallel}$	Δ_b	Δ_1
(LP)	$\frac{r+2}{r+1}$	$\frac{2+r}{2+2r}$		
(P)	4/3	2/3	$\frac{2-r}{3}$	
(C)	4/3	2/3	$\frac{2-r}{3}$	1/3

b) $0<r<2$

For $r<2$, the transition (P) and (C) still have to be distinguished. The phase boundary (P^-), for instance, is still at $B=0$ in $d=2$ but its endpoint (C^-) lies at $h_1>0$ (cf. Fig. 1). The surface exponents for (LP), (P), and (C) with $0<r<2$ are given in Table 2b.

In MF theory, the transition (LD) (see Fig. 1) is discontinuous. At (LD), the potential $V(\ell)$ has the form shown in Fig. 2. The first-order transition occurs in MF theory since the global minimum of $V(\ell)$ changes from $\ell=\tilde{\ell}$ to $\ell=\infty$. In contrast, if $V(\ell)$ of Fig. 2 is used as a "quantum-mechanical" potential, its ground state will tunnel through the potential hill between $\ell=\tilde{\ell}$ and $\ell=\infty$. This implies that all transitions have to be continuous in $d=2$ even in the presence of long-range surface fields.

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References

1. Lipowsky, R.: Z. Phys. B - Condensed Matter **55**, 335 (1984)
2. Lipowsky, R.: J. Appl. Phys. **55**, 2485 (1984)
3. Hauge, E.H., Schick, M.: Phys. Rev. B**27**, 4288 (1983)
4. Meister, T., Müller-Krumbhaar, H.: Phys. Rev. Lett. **51**, 1780 (1983)
5. Kroll, D.M.: Private communication
6. Privman, V.: Private communication
7. Nightingale, M.P., Saam, W.F., Schick, M.: Phys. Rev. Lett. **51**, 1275 (1983)
8. Abramowitz, M., Stegun, I.A.: Handbook of mathematical functions. 5th Edn. New York: Dover Publications
9. Pandit, R., Schick, M., Wortis, M.: Phys. Rev. B**26**, 5112 (1982)
10. Lipowsky, R.: Phys. Rev. Lett. (in press)
11. Amit, D.J.: Field theory, the renormalization group, and critical phenomena. New York: McGraw Hill 1978
12. Lipowsky, R.: Z. Phys. B - Condensed Matter **51**, 165 (1983)
13. Kroll, D.M., Lipowsky, R.: Phys. Rev. B**28**, 5273 (1983)

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