

Critical Wetting in Systems with Long-Range Forces

In a recent Letter,¹ Nightingale, Saam, and Schick studied wetting transitions in three dimensions in the framework of interface models. The pinning potential for the interface coordinate h was taken as

$$V(h) = \text{const} - Ah^{-2}, \quad h > 0, \quad (1)$$

with a hard wall for $h < 0$. It was found that no depinning transition occurs for (1) and $A > 0$. A continuous transition was only found as $A \rightarrow 0^+$. Thus, *critical wetting cannot occur for model (1)*.¹

The potential (1) is related to the effective interaction between the substrate and the interface. For temperature $T=0$, this interaction is given by the difference between the interactions of the adatoms with the real substrate and with a hypothetical

$$F(m) = \int_0^\infty dz \{m(z)\chi(z,m) - u(z)m(z) - \ln \cosh[2\chi(z,m)]\}, \quad (3)$$

with

$$\chi(z,m) = \int_0^\infty dz' K(z-z')m(z'), \quad (4)$$

$K(z-z')$ is the interaction between the adatoms due to the long-range van der Waals forces after integration over lateral directions. We take $K(x) = (J/T)\{1+(x/a)^2\}^{-2}$ where J and a are the overall strength and the range of this interaction. The effective substrate potential $u(z)$ in (3) is taken to be

$$u(z) \rightarrow (B/T)z^{-3}, \quad z \rightarrow \infty \quad (5)$$

with $B > 0$.

The model (3)-(5) has a bulk critical point at $T_c = Ja/\pi$. As $T \rightarrow T_c$, the bulk order parameter behaves as $m_\pm = \pm M = \pm(3t)^{1/2}$ with $t = 1 - T/T_c$. In the following, "liquid" and "gas" mean $m > 0$ and $m < 0$, respectively. The substrate potential (5) can induce a liquid layer of thickness h near the surface while the bulk is still a gas. The free energy of this film is $F(m(z)) - F(m_-)$ where $m(z)$ solves $\delta F/\delta m = 0$. For large h ,

$$\bar{m}(z) = m_+ \theta(h-z) + m_- \theta(z-h) \quad (6)$$

should be a reasonable approximation for $m(z)$. Thus, the free energy of a thick film should be approximately equal to $F(\bar{m}(z)) - F(m_-) = V(h)$. If (6) is inserted into (3)-(5), one finds

$$V(h) = \text{const} + (M/T)\{B - \frac{2}{3}Ja^4M + O(M^3)\}h^{-2}$$

for large h . Thus, the phase boundary for critical wetting is

$$B^* = \frac{2}{3}Ja^4M + O(M^3) \rightarrow 2Ja^4t^{1/2}/\sqrt{3}$$

substrate composed of adatoms.² For general T , however, $V(h)$ is given by the local free-energy density of a rigid interface located a distance h from the substrate. In this Comment, we argue that, for large h , this free energy behaves as

$$V(h) = \text{const} - \{A + A^*(t)\}h^{-2}, \quad (2)$$

where A^* depends on the reduced temperature $t = 1 - T/T_c$. For model (2), critical wetting occurs at $A = -A^*(t)$.³

The potential (2) is obtained as follows. We start from the semi-infinite Ising model defined in Eq. (10) of Ref. 2. Using the standard procedure of a Gaussian transformation,⁴ we obtain a continuum model for the order-parameter field ϕ . Since we will only study mean-field theory, we put $\phi = m(z)$. z is the distance from the substrate surface. This leads to the Landau-Ginzburg free energy

as $T \rightarrow T_c$. With $A = -MB/T$ and $A^* = MB^*/T$, one obtains (2).

Finally, note that (2) contains only the leading terms of $V(h)$ for large h . For short-range forces, next-to-leading terms are important in order to determine the complete phase diagram.⁵ They are also necessary in order to explain the results of van der Waals theory for systems with long-range forces⁶ as will be discussed elsewhere.

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Received 2 April 1984

PACS numbers: 68.10.Cr, 64.60.Fr, 68.55.+b

¹M. P. Nightingale, W. F. Saam, and M. Schick, Phys. Rev. Lett. **51**, 1275 (1983).

²R. Pandit, M. Schick, and M. Wortis, Phys. Rev. B **26**, 5112 (1982).

³In Ref. 1, a short-range potential was included by $V(h) \equiv 0$ for small h . In this case, critical wetting is found if the constant in (2) is negative near T_c .

⁴D. J. Amit, *Field Theory, The Renormalization Group, and Critical Phenomena* (McGraw-Hill, New York, 1978).

⁵R. Lipowsky, D. M. Kroll, and R. K. P. Zia, Phys. Rev. B **27**, 4499 (1983); R. Lipowsky, to be published.

⁶D. M. Kroll and T. Meister, to be published.