Two Scaling Regimes for Complete Wetting by Critical Layers

In a recent Letter,[1] Nightingale and Indekeu used the scaling theory of Fisher and de Gennes[2] for critical adsorption in order to investigate complete wetting by critical layers. The same problem was studied by Lipowsky and Seifert[3] within the framework of Ginzburg-Landau models. It is argued in this Comment that the results of these two studies can be combined to yield a unified picture for such wetting phenomena in systems with short-range forces. This picture provides some further justification for the basic assumption of Nightingale and Indekeu that interfacial fluctuations do not change the critical effects at wetting.

Consider a system where a (multi)critical phase can coexist with a normal phase in the presence of a solid wall. At coexistence, the (multi)critical phase is assumed to wet this wall completely. One may then study how the wetting layer builds up as one approaches the phase boundary of the two phases. The scaling field which governs this approach will be denoted by h. For adsorption, h is proportional to $\ln(p^*/p)$ where p is the pressure and p^* is its value at coexistence. For gravity-thinned layers, h is proportional to the thickness L of the intermediate liquid.[4] For systems with short-range forces, the thickness $\hat{l}$ of the wetting layer diverges as[3]

$$\hat{l} \propto h^{-\mu}, \quad \mu = \tfrac{1}{2}(1-1/q) \tag{1}$$

within mean-field theory which is valid above the upper critical dimension[3]

$$d^*(q) = 2q/(q-1), \tag{2}$$

where $q=2$ and 3 for critical and tricritical layers, respectively. For $d<d^*(q)$, the critical fluctuations within the wetting layer will change the critical exponent μ. In order to find its value, let us consider a different situation, that is, complete wetting by noncritical layers in systems with long-range forces. In this case, the critical exponents have the following properties[5]: They are continuous across the corresponding upper critical dimension d^*; and (2) for $d<d^*$, they depend only on the dimensionality. It is plausible to assume that those two properties are also valid for the complete wetting phenomena studied here. It then follows from Eqs. (1) and (2) that

$$\mu = 1/d, \quad d<d^*(q). \tag{3}$$

Exactly the same value for μ follows from the repulsive finite-size interaction between the two interfaces of the critical layer which has been considered by Nightingale and Indekeu![1]

The above assumption may also be used in order to discuss the behavior of the correlation lengths $\xi_{\parallel} \propto h^{-\nu_{\parallel}}$ and $\xi_{\perp} \propto h^{-\nu_{\perp}}$ which govern the interfacial fluctuations.[3] This leads to

$$\nu_{\parallel} = \begin{cases} \tfrac{1}{4}(3-1/q), & d>d^*, \\ \tfrac{1}{2}(d+1)/d, & d<d^*, \end{cases} \tag{4}$$

and

$$\nu_{\perp} = \tfrac{1}{2}(3-d)\nu_{\parallel}, \quad d<3. \tag{5}$$

The critical behavior described so far should not be affected by interfacial fluctuations as long as $\xi_{\perp} \ll \hat{l}$.[5] From (3)–(5), one sees that this inequality is fulfilled for $d>1$. The same conclusion follows if one compares the finite-size interaction of the interfaces with their overall entropy loss as given by Fisher and Fisher.[6] As a consequence, the basic assumption of Nightingale and Indekeu that the interfacial fluctuations do not change the critical behavior is found to be self-consistent for $d>1$.

If the temperature is changed in such a way that the systems considered here move along a line of critical end points, one may encounter a continuous transition from complete to incomplete wetting at some wetting temperature $\overline{T}_W$. The mean-field behavior of those transitions has been discussed in Ref. 3. It is tempting to apply the above line of reasoning also to this case. This leads to $\hat{l} \propto (T-\overline{T}_W)^{-\overline{\mu}}$ with $\overline{\mu} = (q-1)/[1+(q-1)(d-d^*)]$ for $d<d^*(q)$ which seems to indicate an essential singularity in $d=d^{**}=d^*(q)-1/(q-1)$.

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