Lipowsky and Girardet Reply: In our Letter, we studied the shape fluctuations of solidlike membranes by Monte Carlo simulations and scaling arguments [1]. In contrast to previous simulation studies, we used periodic boundary conditions for the fluctuating membrane in order to suppress edge effects [2], and studied a large number of membranes with different elastic moduli. We found (i) that the roughness exponent is $\zeta=\frac{1}{2}$ and (ii) that there is a pronounced crossover between fluidlike shape fluctuations on small scales and solidlike shape fluctuations on large scales. This crossover should be observable for the plasma membrane of red blood cells or for any other membrane which contains a two-dimensional network with a relatively large mesh size [1].

Our result $\zeta=\frac{1}{2}$ was in sharp contrast to previous results on tethered networks which led to the estimate $\zeta=0.65\pm0.05$. In the preceding Comment [3], Abraham now finds that this estimate for tethered networks is strongly reduced if one uses periodic boundary conditions for these membranes as well. Indeed, he now finds $\zeta\simeq0.53$ which is much closer to our value $\zeta=\frac{1}{2}$. Even though the remaining difference between his estimate $\zeta\simeq0.53$ and our prediction $\zeta=\frac{1}{2}$ is quite small, it has important consequences: For $\zeta>\frac{1}{2}$, the shear modulus becomes scale dependent and vanishes on large scales; for $\zeta=\frac{1}{2}$, on the other hand, the shear modulus may stay finite on large scales. We will now give additional arguments which support both our previous result $\zeta=\frac{1}{2}$ *and* the existence of a finite shear modulus.

If one integrates over the lateral phononlike displacement fields [4], one arrives at the effective Hamiltonian

$$\mathcal{H}\{l\}=\int d^2x\{\tfrac{1}{2}\kappa(\nabla^2 l)^2+\tfrac{1}{2}\Upsilon(\tfrac{1}{2}P_{ij}\partial_i l\,\partial_j l)^2+V(l)\} \quad (1)$$

for the transverse displacement field l, where κ and Υ are the bending rigidity and the Young modulus, respectively. The symbol P_{ij} represents the transverse projector and $\partial_i\equiv\partial/\partial x_i$. The external potential $V(l)$ serves as a large-scale cutoff for the shape fluctuations.

In our Letter, we studied the potential $V(l)=\infty$ for $l<0$ and $V(l)=Pl$ for $l\geq0$. Here, it is more convenient to use the harmonic potential $V(l)=\frac{1}{2}Gl^2$. The statistical weight, $\exp[-\mathcal{H}\{l\}/T]$, then depends on the reduced parameters κ/T, G/T, Υ/T and, in general, on the small-scale cutoff, a.

The perturbative loop expansion for the model as defined by (1) can be expressed in terms of Feynman diagrams. The cutoff dependence of these diagrams is embodied in their one-particle irreducible (1PI) parts which represent the vertex functions, $\Gamma^{(n)}$. Any 1PI diagram $\sim\Upsilon^m$, which contributes to the vertex function $\Gamma^{(n)}$, contains $N_V=m$ vertices, $N_I=2m-n/2$ internal lines or propagators, and $N_L=N_I-(m-1)$ number of loops. In momentum space, each vertex carries a projector $\tilde{P}_{ij}(q)=\delta_{ij}-q_iq_j/q^2$ and a factor $\sim q^4$, the propagators are $\sim T/(\kappa q^4+G)$, and each loop leads to one momentum integration, $\int d^2q$. The latter integrations implicitly contain a high-momentum cutoff $\sim 1/a$. Therefore, such a 1PI diagram scales as $\sim(1/a)^s$ with $s=4m-n-4N_I+2N_L=2(1-m)$. Since $s<0$ for $m>1$, the only divergencies which can occur for large $1/a$ must arise from one-loop diagrams with $m=1$. There is only one such diagram which has to be taken into account [4]. This diagram is, however, *finite* even for $1/a=\infty$, since the projector $\tilde{P}_{ij}(q)$ satisfies $\tilde{P}_{ij}(q)q_i=\tilde{P}_{ij}(q)q_j=0$.

Thus, all 1PI diagrams are finite for $a=0$, and the loop expansion is well defined in this limit. Then, the statistical weight depends only on κ/T, G/T, and Υ/T, and dimensional analysis implies a certain scaling form for the roughness $\xi_\perp$:

$$\xi_\perp^2=\langle l^2\rangle=(T/\kappa)(\Upsilon T/\kappa^2)^{4\nu_\perp-1}(G/\kappa)^{-2\nu_\perp}\sim\kappa^{1-6\nu_\perp}. \quad (2)$$

Our Monte Carlo simulations show that *the limit of zero bending rigidity, $\kappa=0$, is attained in a smooth way.* It then follows from (2) that $\nu_\perp=\frac{1}{6}$. Quite generally, a rough membrane consists of an ensemble of humps [5] with a lateral extension $\xi_\parallel$ and a transverse extension $\xi_\perp\sim\xi_\parallel^\zeta$. This implies a fluctuation-induced potential [5] $V_{FL}\sim T/\xi_\parallel^2\sim 1/\xi_\perp^{2/\zeta}$. Minimization of $V_{FL}+V(\xi_\perp)$ then leads to $\xi_\perp\sim 1/G^{\nu_\perp}$ with $\nu_\perp=\zeta/(2+2\zeta)$. Therefore, $\nu_\perp=\frac{1}{6}$ is equivalent to $\zeta=\frac{1}{2}$.

For $\zeta=\frac{1}{2}$, the shear modulus $\sim\Upsilon$ could still vanish on large scales with a weak logarithmic scale dependence [1]. In the present context, this would imply $\Upsilon_{\rm eff}(\xi_\parallel)\sim 1/[\ln(\xi_\parallel/a)]^r$ with $r>0$ for large $\xi_\parallel$. However, such a behavior cannot apply to the limiting case with $a=0$. This indicates that the shear modulus remains finite on large scales in agreement with our Monte Carlo results.

Reinhard Lipowsky and Marc Girardet
Institut für Festkörperforschung, Forschungszentrum Jülich
D-5170 Jülich, Germany

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[1] R. Lipowsky and M. Girardet, Phys. Rev. Lett. **65**, 2893 (1990).

[2] For a $N\times N$ lattice of mesh points, edge or finite-size effects set in as soon as the longitudinal correlation length $\xi_\parallel\geq N/10$; see R. Lipowsky and B. Zielinska, Phys. Rev. Lett. **62**, 1572 (1989). For solidlike membranes, we studied different types of boundary conditions for the lateral displacement fields which had, however, no significant effect on the scaling properties of the transverse displacement field.

[3] F. F. Abraham, preceding Comment, Phys. Rev. Lett. **67**, 1669 (1991).

[4] D. R. Nelson and L. Peliti, J. Phys. (Paris) **48**, 1085 (1987); D. R. Nelson, in *Random Fluctuations and Pattern Growth,* edited by H. E. Stanley and N. Ostrowsky (Kluwer Academic, Dordrecht, 1988).

[5] R. Lipowsky, Phys. Scr. **T29**, 259 (1989); Nature (London) **349**, 475 (1991).