Appendix to "Adhesion of Membranes with Active Stickers"

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In our simulations, the interaction energy for a Monte Carlo step at time t and site i with rescaled separation z_i is $(V(z_i)/\tau)\int_{t-\tau}^{t} n(t')\mathrm{d}t'$. This interaction energy includes an average over the sticker switching process $n(t')$ in the time interval from $t-\tau$ to t. To understand the resonance in the membrane separation observed in our simulations, it is instructive to consider the escape of a single sticker, or single particle, from the dimensionless stochastic potential $v = (u/\tau)\int_0^t n(t')\mathrm{d}t'$ with binding energy $u = U/k_BT$ and $0 < v < \infty$. At $t = 0$, the sticker is 'on'. To determine the average escape rate $k \equiv \langle e^{-v}\rangle$ at time $t = \tau$, we consider the stochastic differential equation $\mathrm{d}v/\mathrm{d}t = (u/\tau)n(t)$ and construct the corresponding Fokker-Planck equations [1]

$$\frac{\partial P_0}{\partial t} = \omega_- P_1 - \omega_+ P_0 \qquad (1)$$

$$\frac{\partial P_1}{\partial t} = \omega_+ P_0 - \omega_- P_1 - \frac{u}{\tau}\frac{\partial P_1}{\partial v} \qquad (2)$$

Here, $P_1(v,t)$ and $P_0(v,t)$ are the probability distributions for $n(t) = 1$ and $n(t) = 0$, respectively. Multiplying each of these equations with e^{-v} and integrating over v leads to the ordinary first-order differential equations

$$\dot{b}_1 = \omega_+ b_0 - (\omega_- + u/\tau)b_1\,, \quad \dot{b}_0 = \omega_- b_1 - \omega_+ b_0 \qquad (3)$$

for the two auxiliary variables $b_1(t) = \int e^{-v}P_1(v,t)\mathrm{d}v$ and $b_0(t) = \int e^{-v}P_0(v,t)\mathrm{d}v$. In terms of these auxiliary variables, the average escape rate is $k = \langle e^{-v}\rangle = b_1(\tau) + b_0(\tau)$. For the initial condition $n(0) = 1$, and thus $b_1(0) = 1$ and $b_0(0) = 0$, we obtain

$$k = \left(\cosh\Lambda - \frac{u - 2\omega\tau}{2\Lambda}\sinh\Lambda\right)\exp\left[-\frac{u + 2\omega\tau}{2}\right] \qquad (4)$$

with $\Lambda = \frac{1}{2}\sqrt{(u + 2\omega\tau)^2 - 8u\omega\tau X}$.

For the fraction $X = 0.5$, the escape rate k as a function of ω is shown in Fig. 4. In the asymptotic limit of slow switching or small ω, the sticker has a high probability to be still in the initial on state. The average potential of the sticker then is identical with its binding energy u, and the escape rate simply is e^{-u}. In the limit of fast switching or large ω, the sticker will switch very often between the states. The average potential then is $Xu = u/2$, and the escape rate $e^{-Xu} = e^{-u/2}$. But at the intermediate switching rates, the average potential depends on the switching process and can vary from values close to zero (if the sticker is switched off early in the interval τ and remains off) to the value u (if the sticker remains in the on state). The resonance effect at the intermediate switching rates can be understood from the contribution of switching processes with small average potential v to the escape rate $k = \langle e^{-v}\rangle$. The effect is closely related to the 'resonant activation' of particles in potentials with a fluctuating barrier [2].

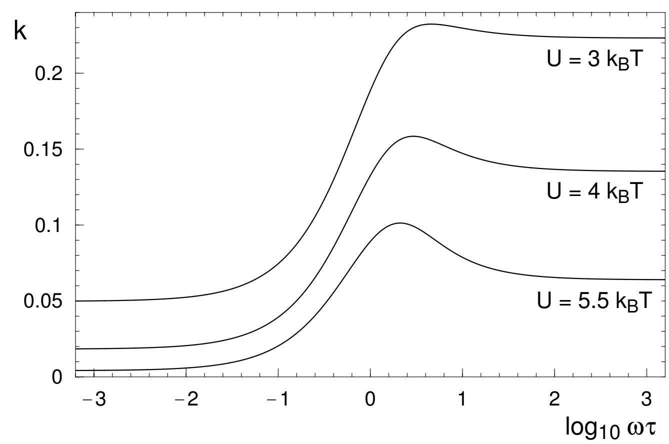

FIG. 1: Escape rate k for a single sticker as a function of the mean switching rate ω at the average fraction $X = 0.5$ of the on state and several values of the sticker binding energy U, as given by eq. (4).

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